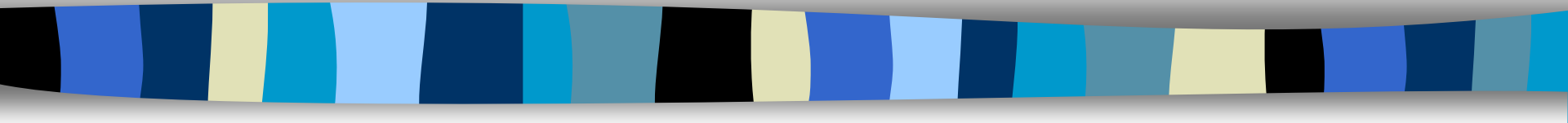


- 2. Speech and voice technologies
 - DSP reminder
 - The speech signal
 - Speech Technologies
 - Overview
 - Modelling (parameters, models)

Digital Signal Processing Reminder

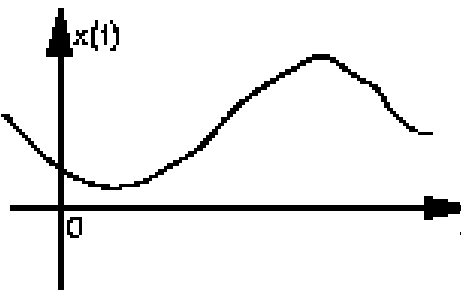
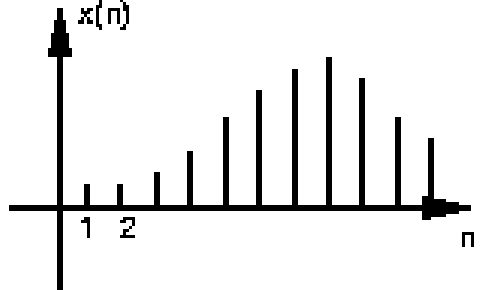
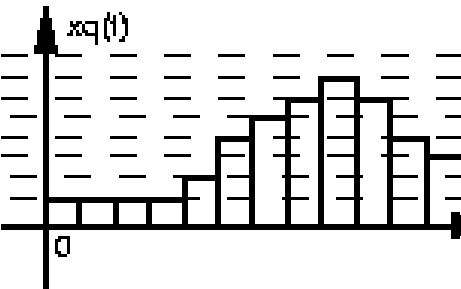
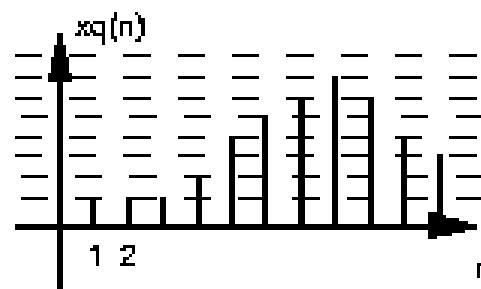




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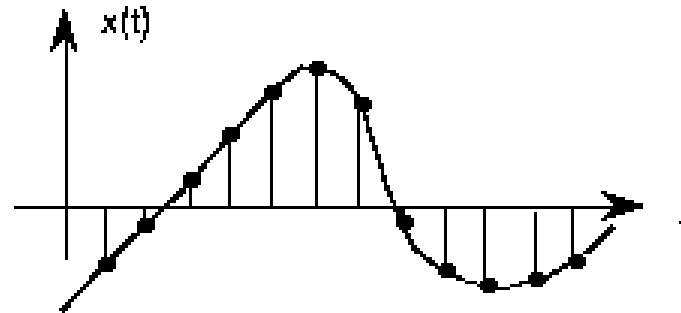
Digital / Analogic Signals

	TEMPS CONTINU	TEMPS DISCRET
AMPLITUDE CONTINUE	 <p>A graph showing a continuous signal $x(t)$ plotted against time t. The vertical axis is labeled $x(t)$ and the horizontal axis is labeled t. The origin is marked with 0. The signal is a smooth, continuous curve that starts at a low value, rises to a peak, and then falls.</p>	 <p>A graph showing a discrete signal $x(n)$ plotted against time n. The vertical axis is labeled $x(n)$ and the horizontal axis is labeled n. The origin is marked with 0. The signal consists of vertical bars at discrete time intervals, with the first two bars labeled 1 and 2, and the last bar labeled n. The bars vary in height, forming a discrete approximation of the continuous signal.</p>
AMPLITUDE DISCRETE	 <p>A graph showing a discrete-amplitude signal $x_q(t)$ plotted against time t. The vertical axis is labeled $x_q(t)$ and the horizontal axis is labeled t. The origin is marked with 0. The signal consists of vertical bars at discrete time intervals, with the first two bars labeled 1 and 2, and the last bar labeled t. The bars are constrained to a set of discrete amplitude levels, indicated by horizontal dashed lines. The bars vary in height, forming a discrete approximation of the continuous signal.</p>	 <p>A graph showing a discrete-amplitude signal $x_q(n)$ plotted against time n. The vertical axis is labeled $x_q(n)$ and the horizontal axis is labeled n. The origin is marked with 0. The signal consists of vertical bars at discrete time intervals, with the first two bars labeled 1 and 2, and the last bar labeled n. The bars are constrained to a set of discrete amplitude levels, indicated by horizontal dashed lines. The bars vary in height, forming a discrete approximation of the continuous signal.</p>

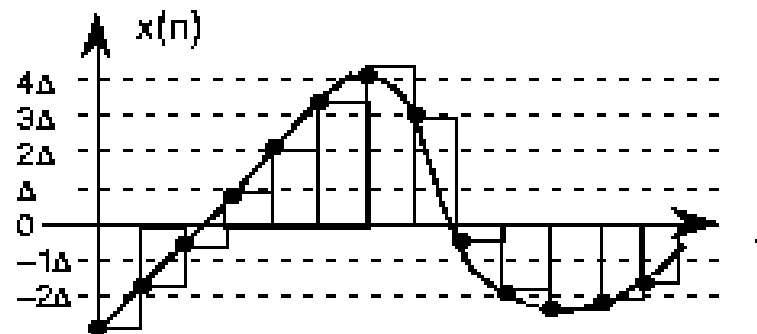
Analog-Digital Conversion

■ Sampling and Quantification

sampling



quantification





Energy of a signal

- *Continuous signal $s(t)$ on $[t_1, t_2]$*

$$W_s(t_1, t_2) = \int_{t_1}^{t_2} s^2(t) dt$$

- *Discrete signal $x(n)$ on $[n_1, n_2]$*

$$W_s(n_1, n_2) = \sum_{n=n_1}^{n_2} s^2(n)$$

SNR : Signal-to-Noise Ratio

- For $x(t)=s(t)+n(t)$

$$SNR = \frac{W_s}{W_n}$$

$$SNR_{dB} = 10 \log_{10} SNR$$

$$SNR_{dB} = 20 \log_{10} \frac{\text{Amplitude}_s}{\text{Amplitude}_n}$$



Fourier Transform - TF

- Spectral representation of signals
- Core mathematical tool in DSP

TF for continuous signals

- $x(t)$ signal
- TF is a function of variable $\omega = 2\pi f$ defined by :

$$F\{x(t)\} = X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t} dt$$

- Inverse transform

$$x(t) = F^{-1}\{X(\omega)\} = \int_{-\infty}^{+\infty} X(\omega)e^{+j\omega t} d\omega$$

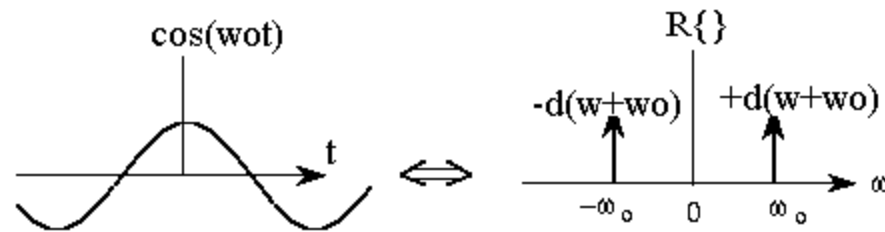
(2-D)

$$F\{g(x, y)\} = G(u, v) = \int_{-\infty-\infty}^{+\infty+\infty} g(x, y) e^{-j(ux+vy)} dx dy$$

$$g(x, y) = F^{-1}\{G(u, v)\} = \int_{-\infty-\infty}^{+\infty+\infty} G(u, v) e^{j(ux+vy)} du dv$$

TF of a periodic signal (cos)

$$\cos(\omega_0 t) \stackrel{TF}{\leftrightarrow} \frac{1}{2} [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$



Time-frequency representation

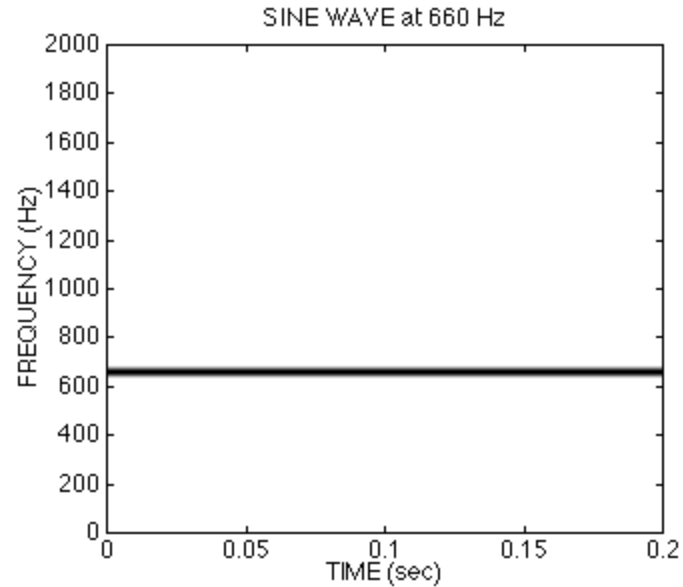
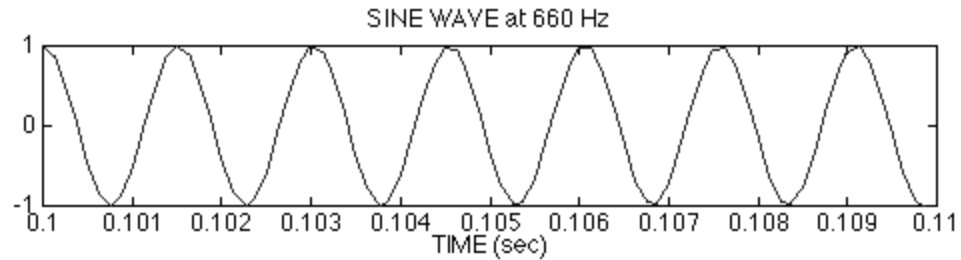




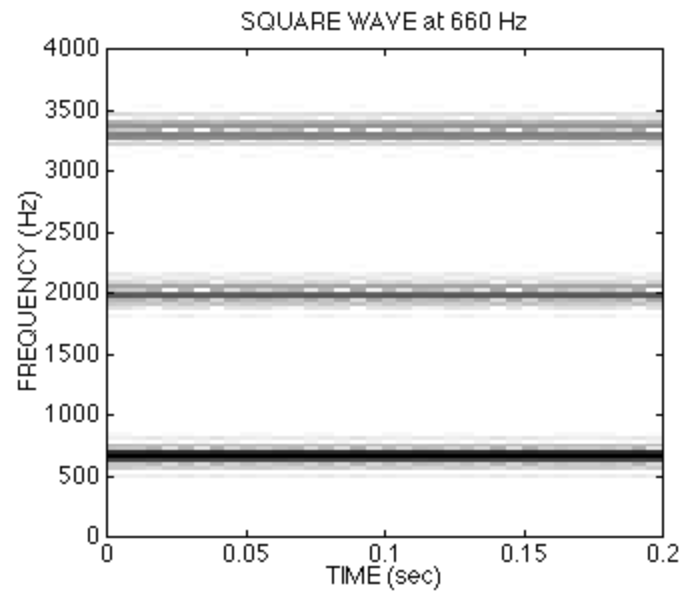
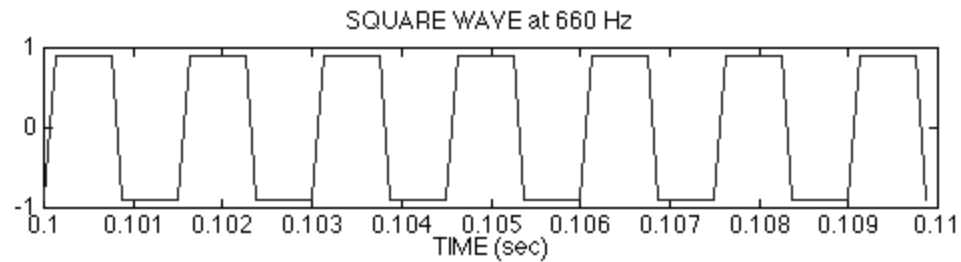
Time-frequency representation Spectrogram

$$S_x(t, f) = \left| \int_{-\infty}^{+\infty} x(s) h^*(s-t) e^{-i2\pi f s} ds \right|^2$$

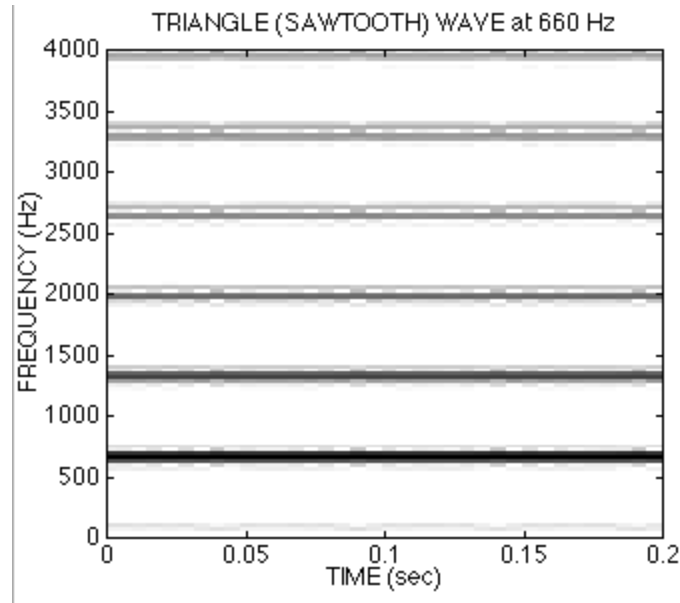
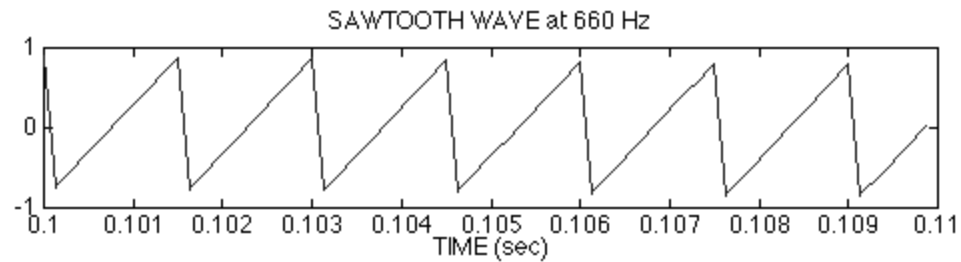
Case of a sinus



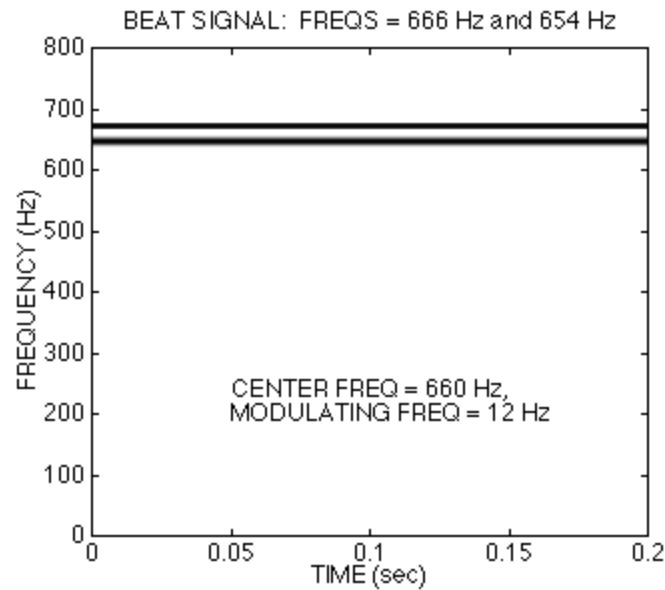
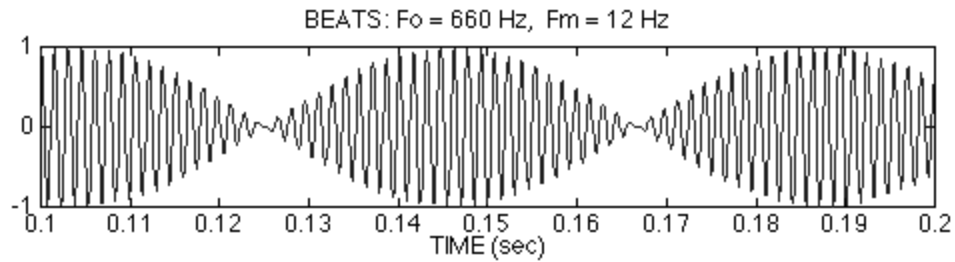
Case of square signal



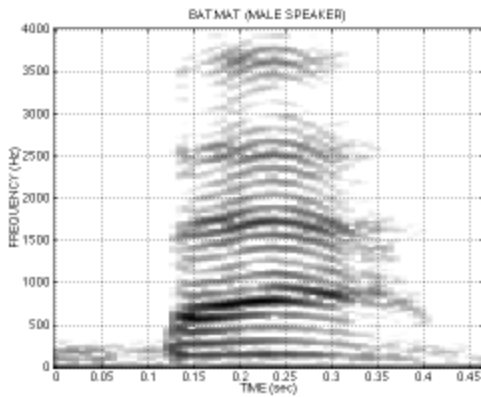
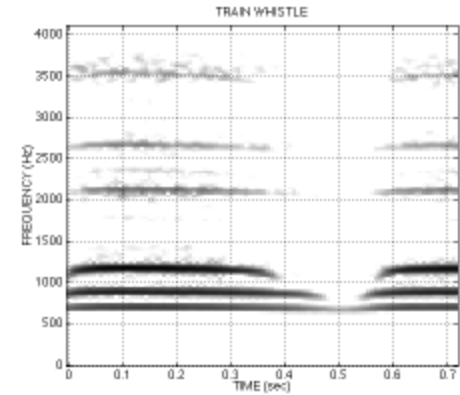
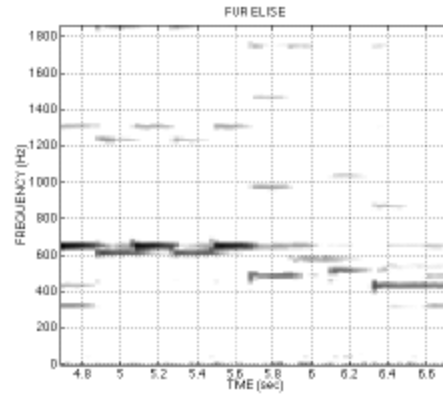
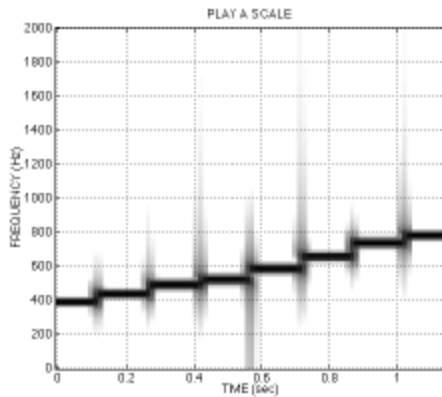
Sawtooth signal



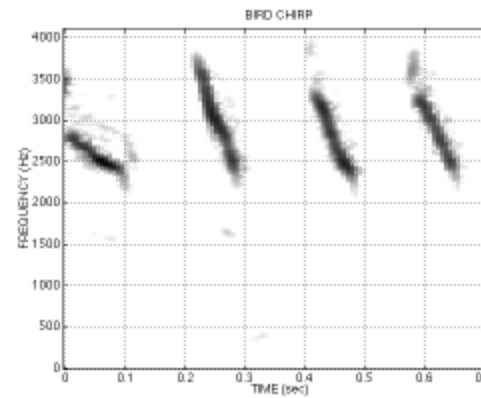
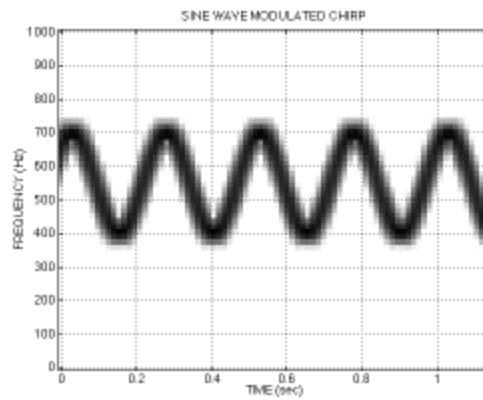
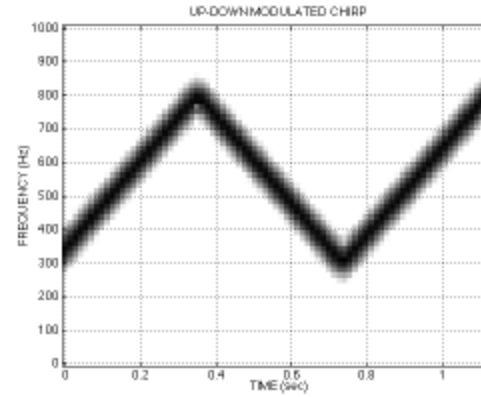
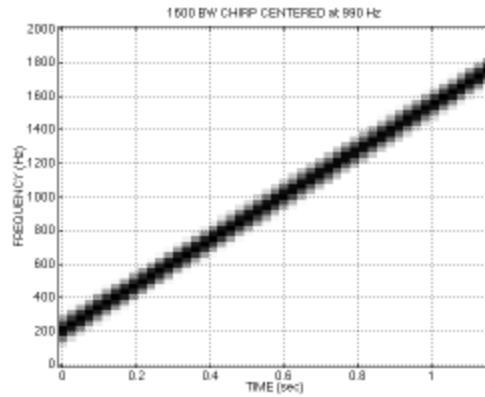
Beat Signals



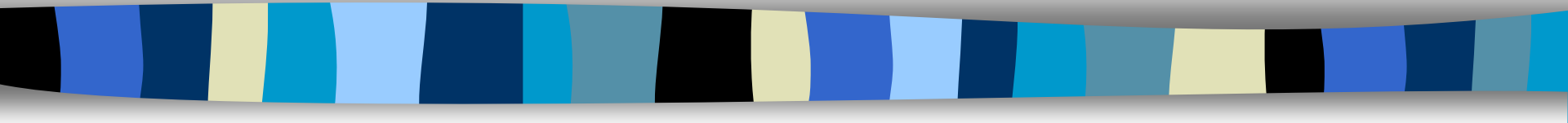
Other examples



Chirps



The Speech Signal





Speech : one modality among others ?

■ From the user point of view

- As output
 - *Visual (2D,3D)*
 - **Sound/Speech**
 - ...
- As input
 - **Spoken command**
 - Gesture
 - ...



Speech in HMI : ergonomic aspects

■ Communication Mean

- natural
- fast and concise
- Hands free
- Gaze can be used for other task
- Helpful for disabled people



But...

- **Critical Situations**
 - degradation of performances
- **Training of the Machine Needed**
 - Might be long, constraints (do you know IBM ViaVoice?)
- **Linguistic constraints**
 - Vocabulary size
 - Syntactic structure
 - Mode (isolated words, connected words, natural spoken language, disfluencies)
- **Human factors**
 - micro + headset => acceptability of the user
 - Privacy issues



Goals

- Satisfaction of the user
- Quality
 - System delay
 - Robustness
 - Need to have plan B (or C, or D...)
- Cost
 - Use of speech must have a limited impact on the costs



Avoid...

- Full speech interface
 - It is just another modality
- Complicated systems
 - ergonomics
- Annoy user
 - Robustness needed



Speech technologies

- Speech compression / coding (*wireless, IP,...*)
- Speech synthesis / recognition for dialog systems (information access...)
- Speaker authentication (voice biometrics)

- More details will be given in my last course...



Speech sounds : phonemes

- Establish distinctive units of meaning
- **Phonemes** are the shortest sound units in speech that allow to distinguish different words
- Examples [p] [b]
 - pas / bas
 - paie / baie
 - pot / beau

French phonemes

TABLEAU I. — *Les phonèmes du français*

Consonnes

[p] paie	[t] taie	[k] quai
[b] baie	[d] dais	[g] gai
[m] mais	[n] nez	[ɲ] gagner
[f] fait	[s] sait	[ʃ] chez
[v] vais	[z] zéro	[ʒ] geai
[w] ouais	[y] huer	[j] yéyé
	[l] lait	[R] raie

Voyelles

[i] lit	[y] lu	[u] loup
[e] les	[ø] leu	[o] lot
[ɛ] lait	[œ] leur	[ɔ] lotte
[a] là	[ə] le	
[ɛ̃] lin	[ɑ̃] lent	[õ] long

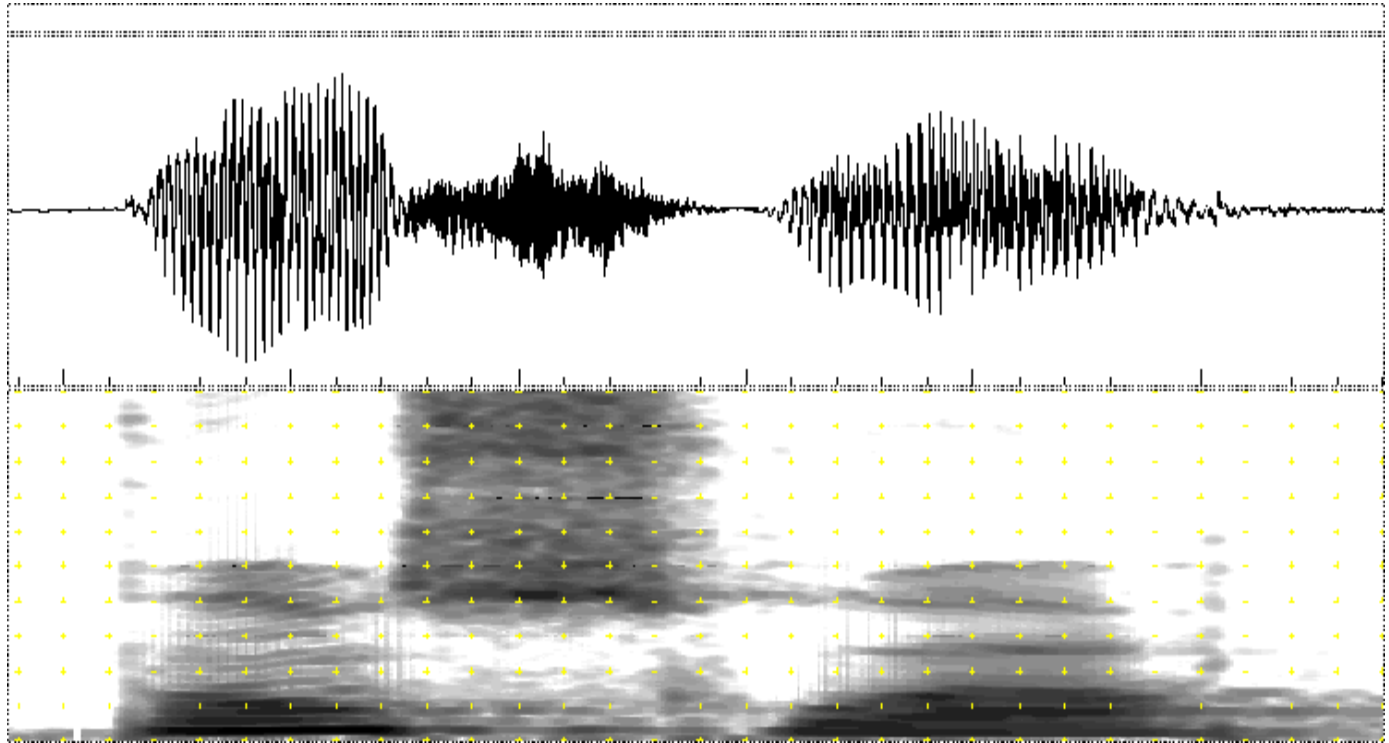
Note : Les distinctions vocaliques [e]-[ɛ], [ø]-[œ] et [o]-[ɔ] ne sont pas faites dans tous les contextes et par tous les locuteurs du français. Par contre, certains locuteurs font aussi des distinctions entre patte et pâte, ([a]-[ɑ]) ainsi qu'entre brin et brun ([ɛ̃]-[œ̃]).

French phonemes

TABLEAU II. — *Classification des phonèmes du français en traits distinctifs*

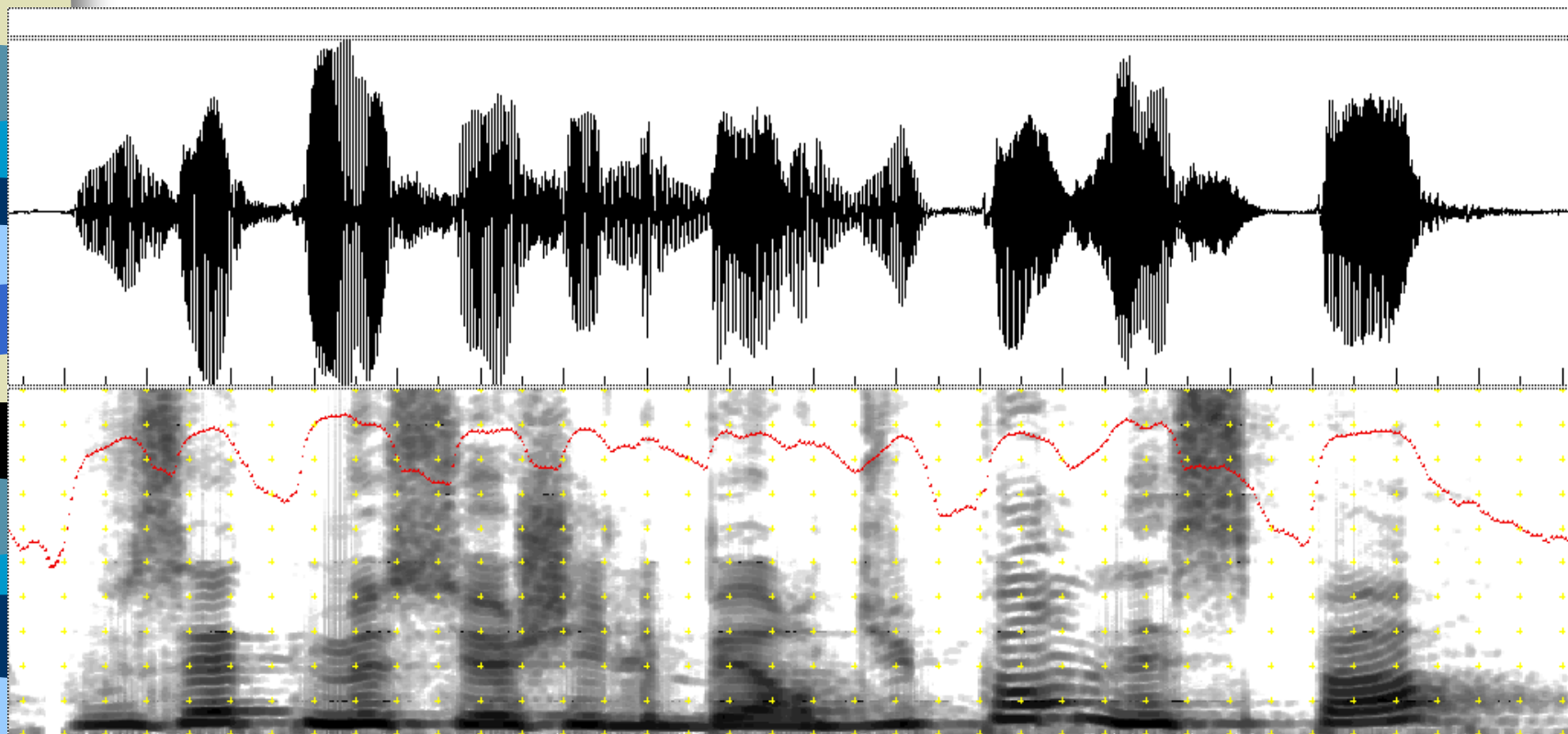
CONSONNES				Lieu d'articulation
Mode d'articulation	Labiales	Dentales	Vélo-palatales	←
↓				
Occlusives				
non voisées	[p]	[t]	[k]	
voisées	[b]	[d]	[g]	
Nasales	[m]	[n]	[ŋ]	
Fricatives				
non voisées	[f]	[s]	[z]	
voisées	[v]	[z]	[ʒ]	
Glissantes	[w]	[y]	[j]	
Liquides		[l]	[R]	
VOYELLES				
Orales	Antérieures		Postérieures	
	Non arrondies		Arrondies	
Fermées	[i]	[y]	[u]	
	[e]	[ø]	[o]	
	[ɛ]	[œ]	[ɔ]	
Ouvertes	[a]			
Nasales	Antérieures		Postérieures	
Fermées	[ɛ̃]		[ɔ̃]	
Ouvertes		[ɑ̃]		

Speech signal



Bonsoir

Speech signal



Vous êtes Monsieur Gilbert Dupont n'est-ce pas ?

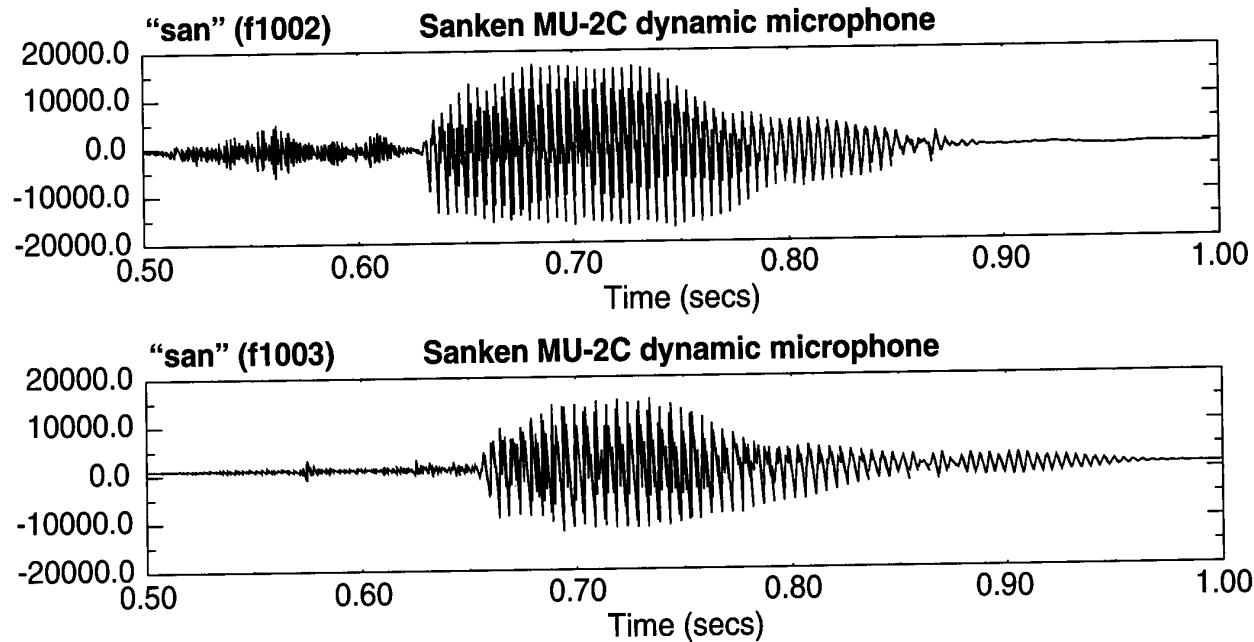


Variability

- First characteristic of speech signals is variability
 - A speaker never pronounces two times the exact same sound
 - Two speakers do not pronounce the same sound the same way
- However, this sound is perfectly recognized by a human listener

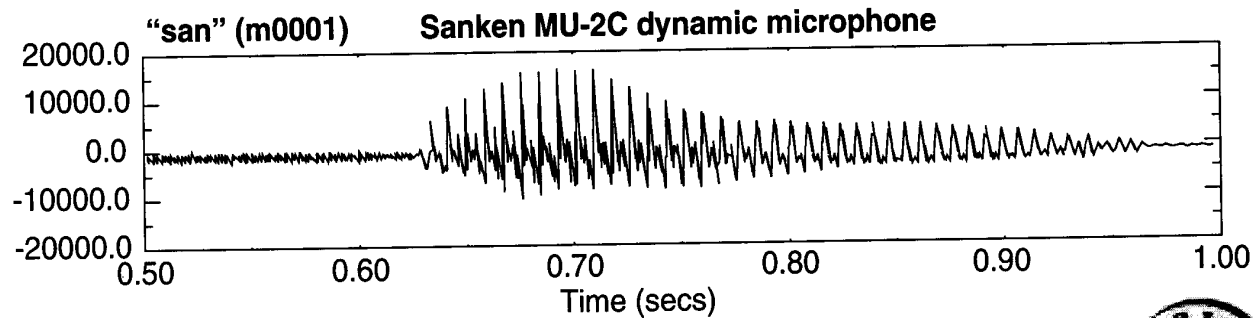
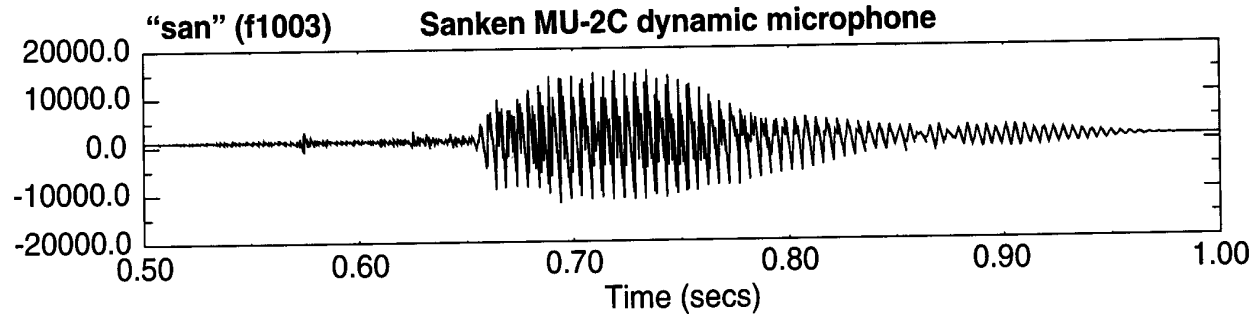
Intra-Speaker variability

Same sounds, same person,
same recording conditions



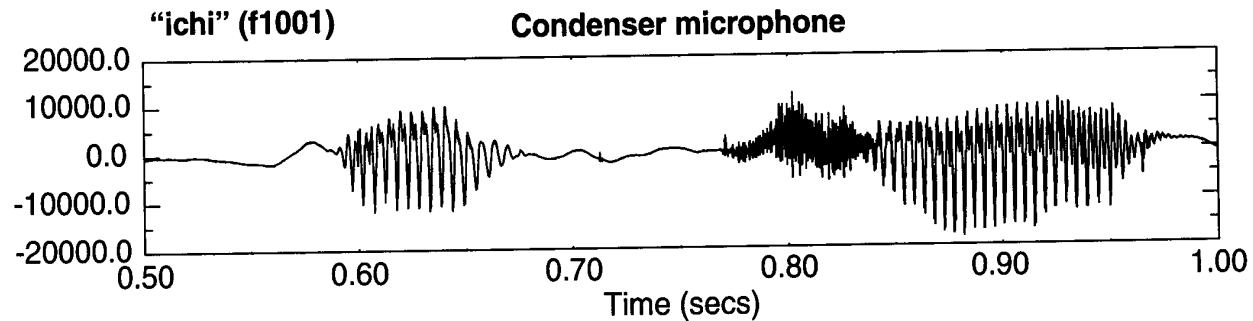
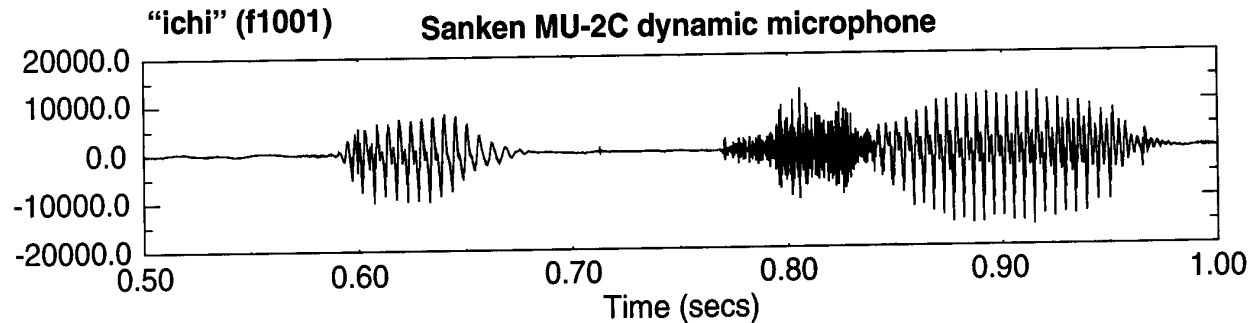
Inter-speaker variability

Same sounds, different persons,
same recording conditions

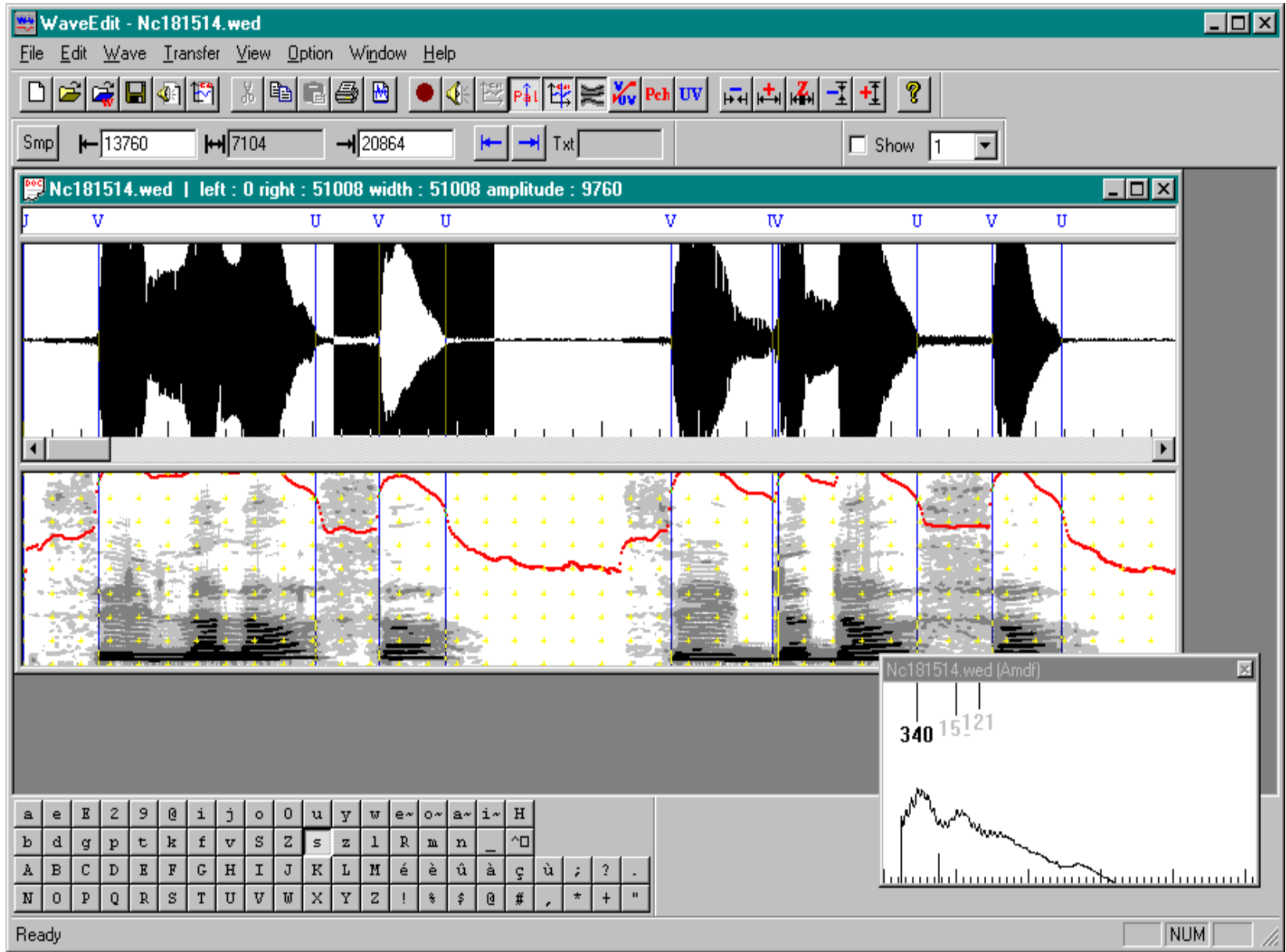


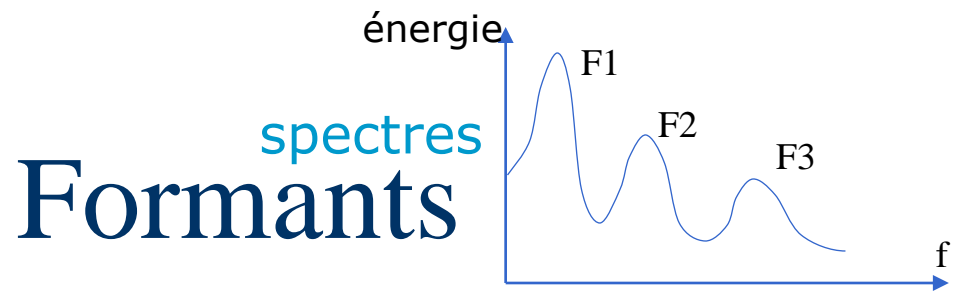
Variability due to recording conditions

Same sounds, same person,
different microphones

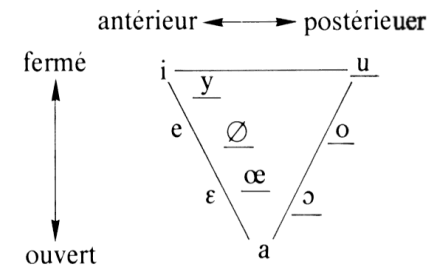
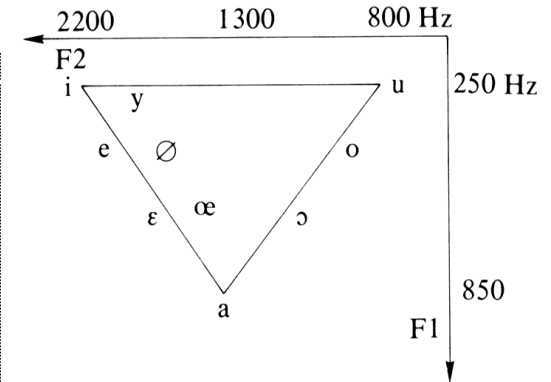
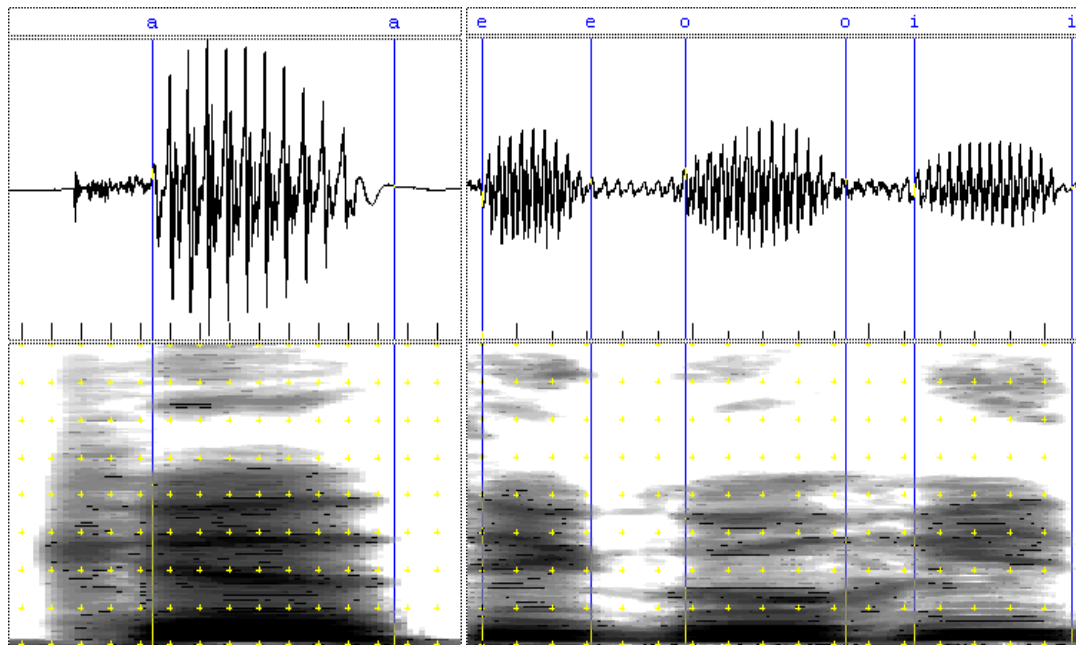


Tools for speech analysis





- The spectral peaks of the sound spectrum $|P(f)|'$ of the voice
- Acoustic resonance of the human vocal tract
- Vocal triangle for vowels



Formants

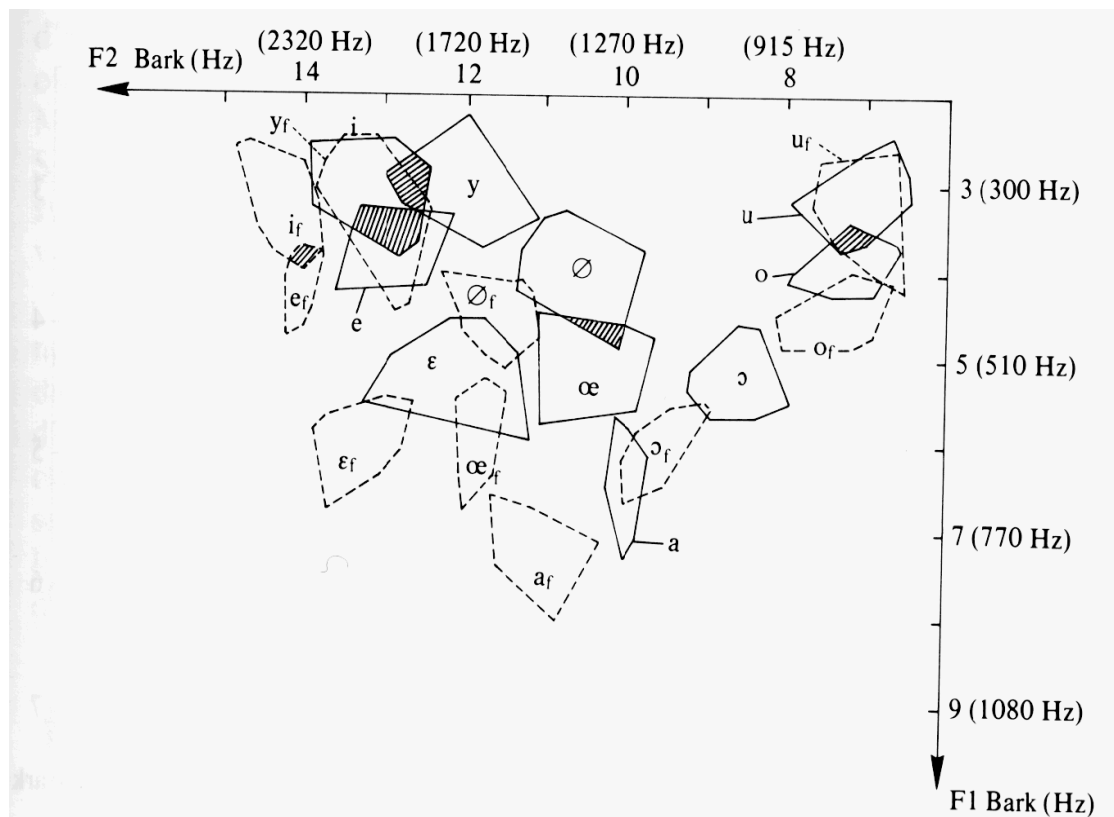


FIG. III.6. — Zones de dispersion des voyelles non nasales du français sur le plan F_1/F_2 (échelle de Bark).

Sujets masculins : ————

Sujets féminins : - - - - -

Les hachures délimitent les zones de recouvrement pour un même sexe.

Variability of speech and speakers

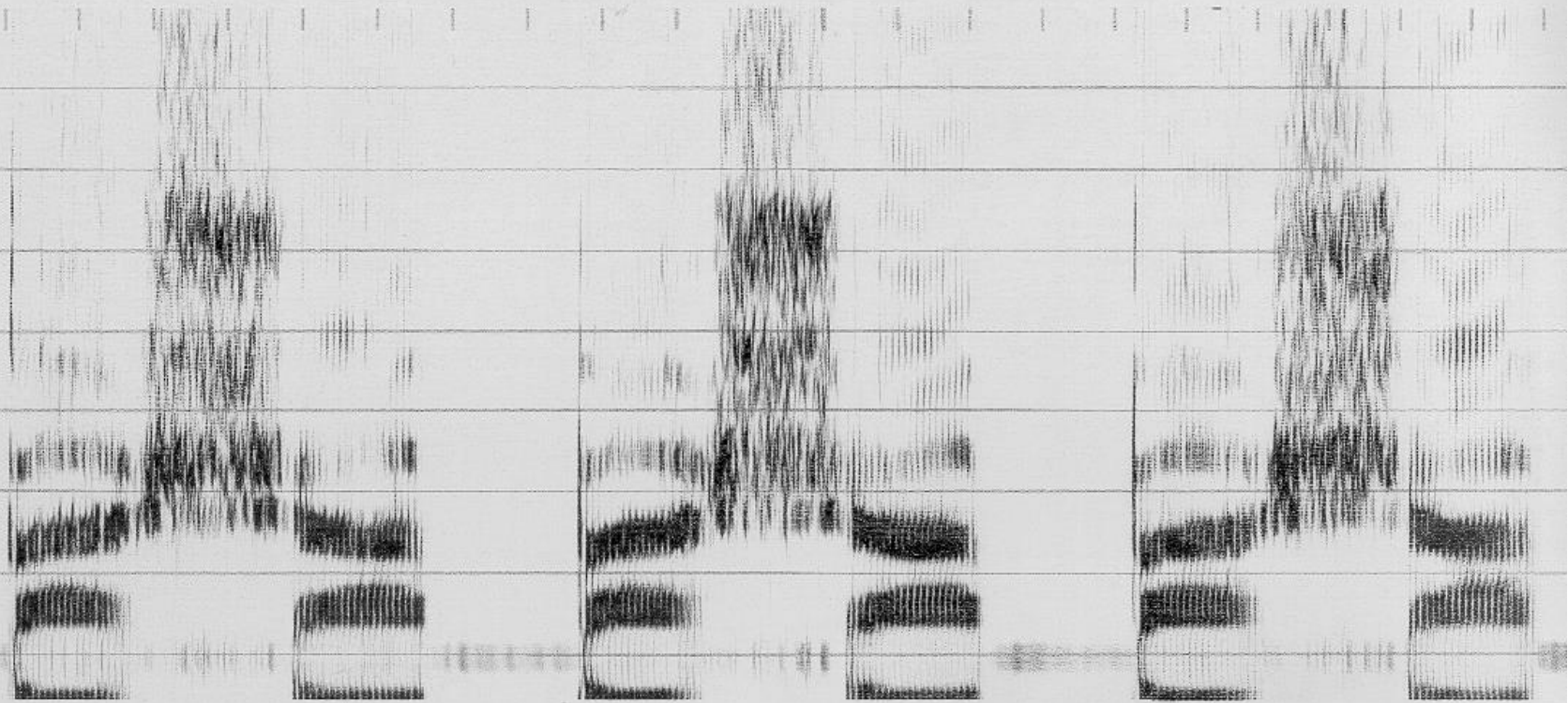
Formants : values for french vowels

TABLEAU III.1. — Valeur en Hz des 4 premières fréquences formantiques. 10 sujets masculins ; 9 sujets féminins — Analyse LPC. Med. : valeur de la médiane ; σ : écart-type ; Ki : coefficient d'écart entre formants masculins et féminins : ($Ki = F1\text{M}/F1\text{F}$). Corpus : [pV] ou [pVR], 2 répétitions par sujet.

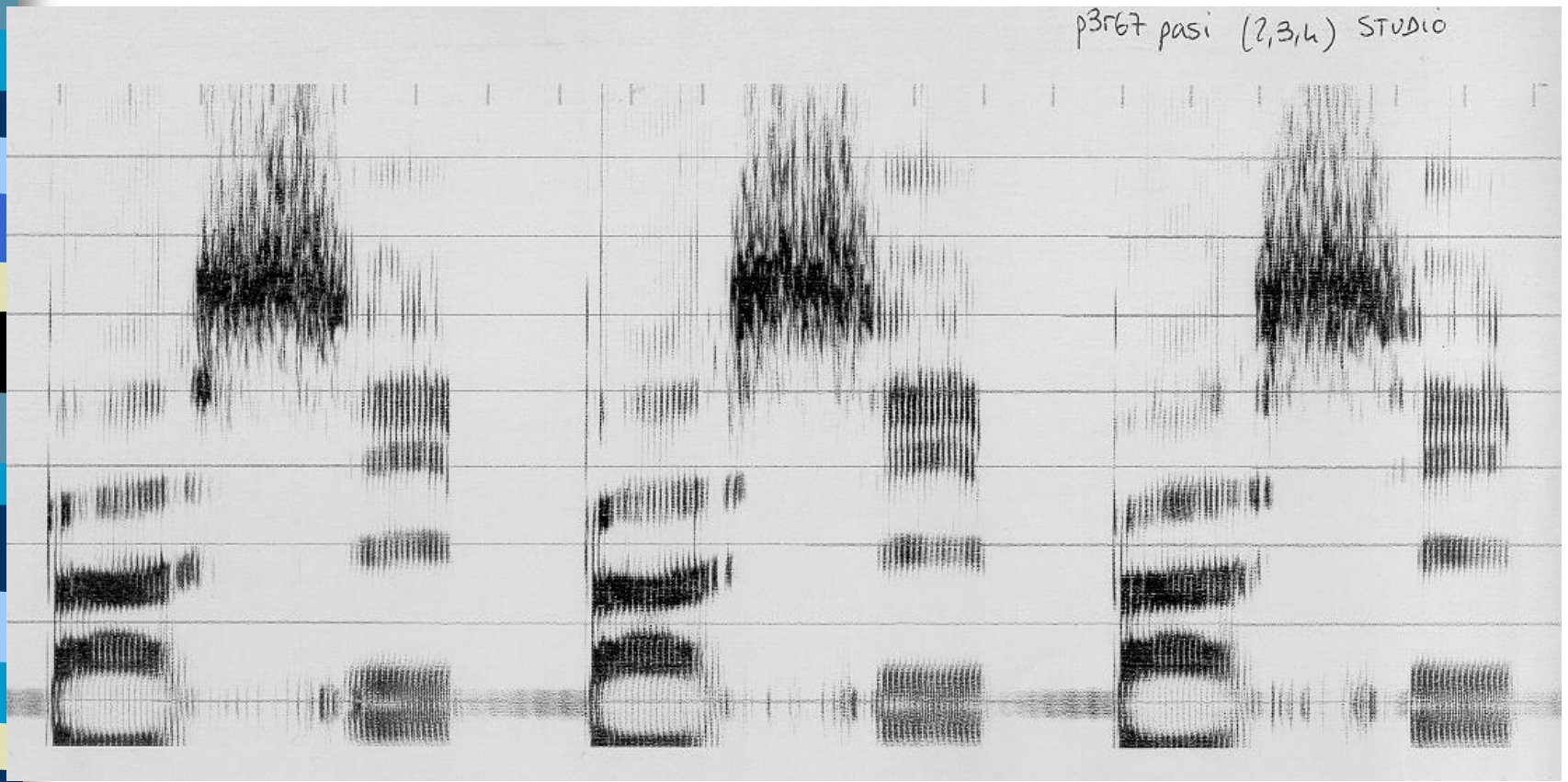
voyelles	Sujets masculins				Sujets féminins					
	F_1	F_2	F_3	F_4	F_1	F_2	F_3	F_4		
i	Med.	308	2064	2976	3407	Hz	306	2456	3389	3966
	σ	34	134	147	208	Hz	42	111	68	169
	Ki	0,99	1,19	1,14	1,16	%				
e	Med.	365	1961	2644	3362		417	2351	3128	4161
	σ	31	119	107	155		31	52	115	121
	Ki	1,14	1,20	1,18	1,24					
ɛ	Med.	530	1718	2558	3300		660	2080	2954	4231
	σ	49	132	103	221		46	108	156	210
	Ki	1,25	1,21	1,15	1,28					
a	Med.	684	1256	2503	3262		788	1503	2737	4143
	σ	47	32	131	155		51	86	174	192
	Ki	1,15	1,20	1,09	1,27					
ɔ	Med.	531	998	2399	3278		634	1180	2690	3950
	σ	39	60	116	155		48	59	198	201
	Ki	1,19	1,18	1,12	1,21					
o	Med.	383	793	2283	3256		461	855	2756	3805
	σ	22	63	126	161		38	73	240	183
	Ki	1,20	1,08	1,21	1,17					
u	Med.	315	764	2027	3118		311	804	2485	3550
	σ	43	59	136	172		43	53	284	197
	Ki	0,99	1,05	1,23	1,14					
y	Med.	300	1750	2120	3145		305	2046	2535	3570
	σ	37	121	182	141		68	124	139	216
	Ki	1,02	1,17	1,20	1,14					
ø	Med.	381	1417	2235	3215		469	1605	2581	4005
	σ	44	106	113	201		36	90	148	168
	Ki	1,23	1,13	1,15	1,25					
œ	Med.	517	1391	2379	3353		647	1690	2753	4038
	σ	42	94	91	149		58	47	155	202
	Ki	1,25	1,21	1,16	1,20					

Spectrogram

p3r66 paʃa (2,3,4) STUDIO

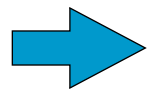


Spectrogram (large band) - 2



Prosody

- Pitch or fundamental frequency
- Voice energy
- Syllable / phoneme duration



Intonation / Voice « melody »



Fundamental frequency f_0

- Vibration of the vocal cords
- Depends on speaker age and sex
 - 100 à 150 Hz for adult male speaker
 - 140 à 240 Hz for adult female speaker
- Can have huge variations for a single speaker
 - Depending on the type of sentence uttered
 - Depending on the emotional / affect of the speaker



Practical lab (optional)

- Speech Analysis Tools
- <http://www-clips.imag.fr/geod/User/laurent.besacier/NEW-TPs/TP-CL/tp7.html>

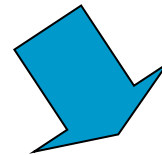
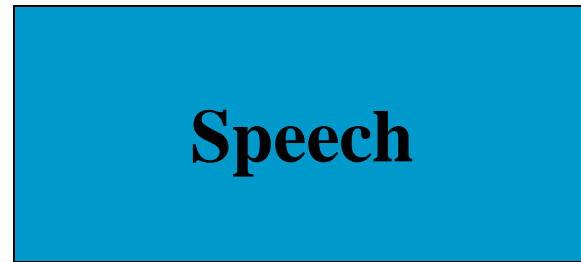
Speech technologies



Overview

Modelling (parameters, stochastic models HMMs)

Speech, a source of informations

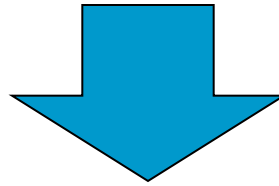


Linguistic informations
(what is uttered)

Extra-linguistic info.
(speaker, language, speaker
state)

Linguistic Informations

- What is uttered by the speaker....



Automatic Speech Recognition (ASR)



Different levels of difficulty

- **Number of speakers** : systems mono-speakers ...until multi-speakers
- **Vocabulary size**
- **Transmission channel** : «direct mic. », téléphone, mobile phone, VoIP



Different levels of difficulty

- **Acoustic Environment** : quiet, normal (officeroom), noisy (train station, street), extreme (plane cockpit)

- **Speaking style** : digits, isolated words, connected words, continuous speech (read, spontaneous)

- 1 person or conversation



Applications

- **Services** (vocal servers)
- **Vocal terminals** (on site)
- **Transports** (vocal commands help, command for navigation system, EVV)
- **Language learning**
- **Dictation**
- **Information retrieval**
- **Control / vocal commands**



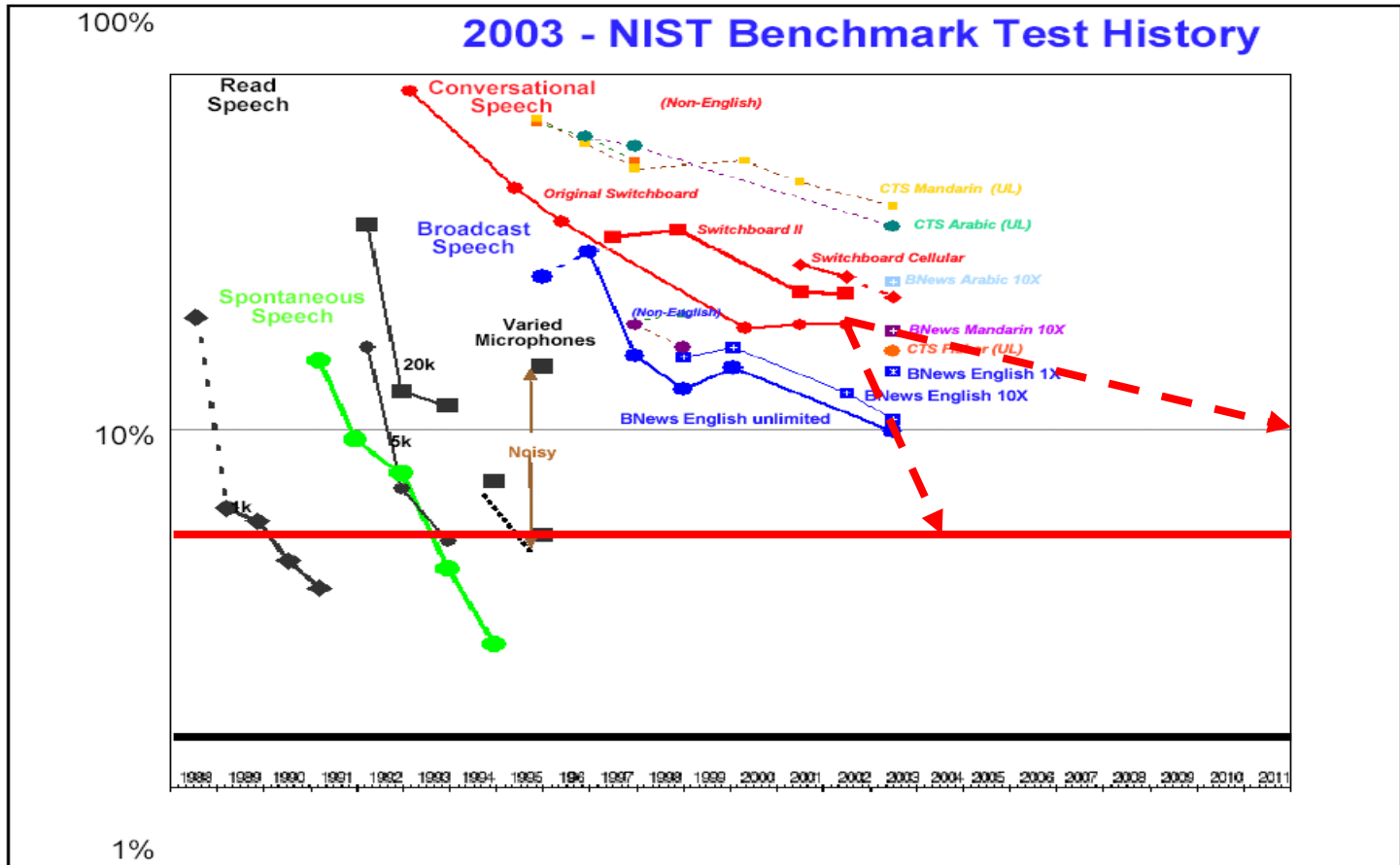
Where we are...

- **Best systems achieve***
 - ~10-12% WER for English on European Parliament Speeches or Broadcast News Data !
 - ~20% WER for English on broadcast or telephone conversations
- **Large Improvements over the years**

See DARPA & NIST evaluations...

*sources: TCSTAR & GALE projects

Where we are...



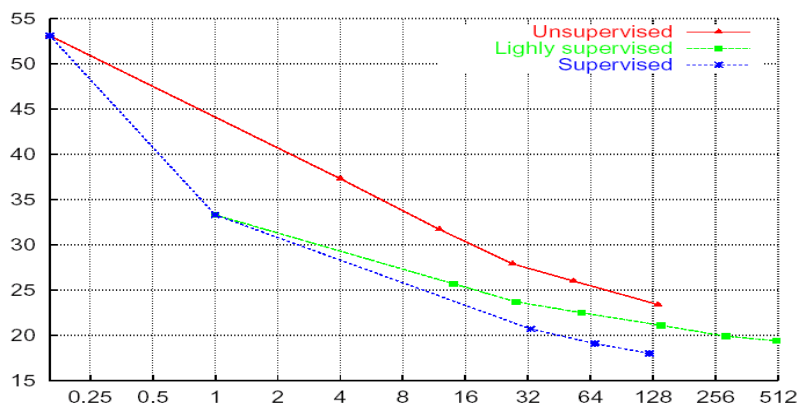


Where we are...

- Improvements over the last 15 years mostly due to...
 - Better modeling : discriminant approaches (MMI,MPE), tying (mixtures, states)
 - Adaptation techniques (MAP,MLLR,VTLN)
 - Computational power : for multipass decoding and multiengine approaches (ROVER)
 - And last but not least...

Where we are...

- **More Data !!**
- ***“There’s no data like more data”***, Robert L. Mercer



From LIMSI, Lamel (2002)

Training (hrs)	141	297	602	843
WER(%)	17.2	15.4	14.7	14.5

From RT03 (BBN)

Where we go...

■ Evolution of the domain

- ‘Simple’ Transcription → Rich Transcription
- Controlled Audio Stream → Continuous Audio Stream
- One sensor → Multiple sensors
- Monolingual → Multilingual
- Audio only → Multimodal

■ Increasing difficulty of the tasks

Dictation

*Broadcast news
Transcription*

*Meetings
Smart rooms*





Human versus machine

Task	Machine	Human
Connected digits	0.72%	0.009%
Letters	5.0%	1.60%
Transactional speech	3.6%	0.10%
Dictation	7.2%	0.9%
Conversational telephone	43.0%	5.0%

Source: R. Lippmann, *Speech Communication*, 1997



Limits and open issues

- Rich Transcription
 - Mark speaker turns, disfluencies, ...
- Continuous Audio Flaw
 - Need for sentence breaks, punctuation, ...
- Multiple sensors
- Multilingual
 - Portability to new languages, non native speakers
- Multimodal
 - Multiple data streams, asynchronism

Statistical modelling

$$\hat{P}(Y|X)$$

Sequence of acoustic observations

- *Signal frames*
- *Filterbank coefficients*
- *Cepstral coefficients*
- *Time-frequency principal components*
- ...

Sound object (or class) hypothesis

- *Sound type (speech / music / ...)*
- *speaker / language / channel*
- *phone / syllable / word*
- *Sound event (jingle)*
- *Past or future of a break (ex: speaker change)*
- ...

→ Generic Approach



Speech parameters

- Acoustic parameters extracted from speech
 - LPC (Linear Predictive Coefficients)
 - MFCC (Mel Frequency Cepstral Coefficients)
 - FilterBanks

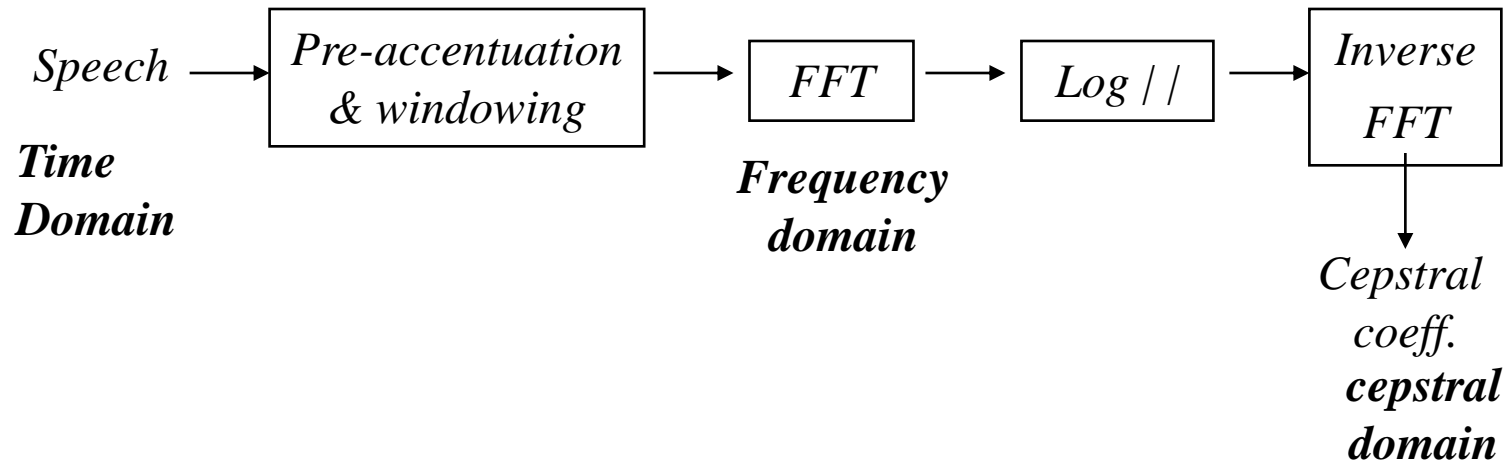


Speech parameters

- Mostly for automatic speech recognition and speech compression
 - Spectral analysis
 - Cepstral analysis
 - Linear prediction
- Also used
 - Prosodic information (fundamental frequency, energy features, duration)

Acoustic parameters

- Filterbank coefficients : signal energy in different frequency bands
- Cepstral coefficients



Acoustic parameters

■ LPC (Linear Predictive Coding)

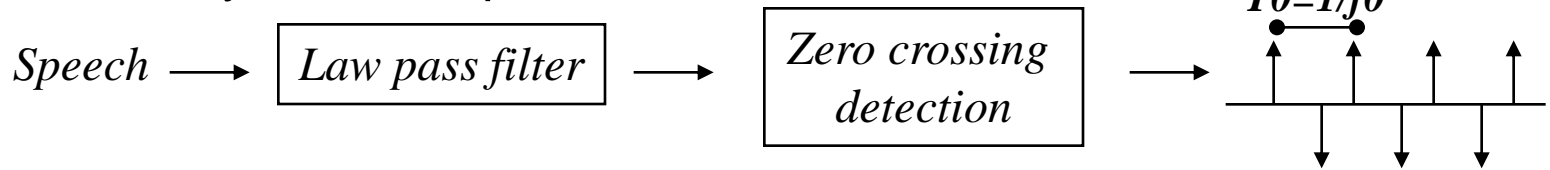
- A sample is predicted as a weighted sum of preceding samples

$$\hat{s}_n = \sum_{i=1}^p a_i s_{n-i}$$

- p is the model order
- a_i = linear prediction coefficients
- different methods to predict this coeff. (levinson-durbin algo.)

Acoustic parameters

- Fundamental frequency (pitch or f_0) :
 - analyseurs temporels



- Problem with pitch : large variability, fine estimation is difficult...

Statistical modelling

$$\hat{P}(Y|X)$$

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→ Generic Approach

Bayes

- x : observation (signal)
- c_i : class to be recognized

$$c^* = \operatorname{argmax}_i p(c_i / x) = \operatorname{argmax}_i \frac{p(x / c_i) \cdot P(c_i)}{p(x)} \approx \operatorname{argmax}_i p(x / c_i) \cdot P(c_i)$$

- Automatic Speech Recognition (ASR)

$$w^* = \operatorname{argmax}_i \frac{p(x / w_i) \cdot P(w_i)}{p(x)} = \operatorname{argmax}_i p(x / w_i) \cdot P(w_i)$$

Acoustic model
↑
↓
Language model

Bayes

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Acoustic model
↑
Language model
↓

- Statistical Machine Translation (SMT)

$$e^* = \operatorname{argmax}_i \frac{p(f / e_i) \cdot P(e_i)}{p(f)} = \operatorname{argmax}_i p(f / e_i) \cdot P(e_i)$$

Translation model
↑
Language model
↓



Phone (Acoustic) Models

- Generally, the acoustic units modeled are phonemes rather than words
 - Exemple : ~40 phone models for french
- To calculate $p(x/w_i)$ an acoustic model, as well as a pronunciation dictionary are needed



Context Dependent vs. Context Independent Models

- **Independent** : each unit is modeled independently of the others
- **Dependent** : different models for a same phone unit according to the left-right context
- **triphones** : only nearest left and right phonemes are considered

- => due to **coarticulation**
- => Problem : corpora never big enough to estimate robust models

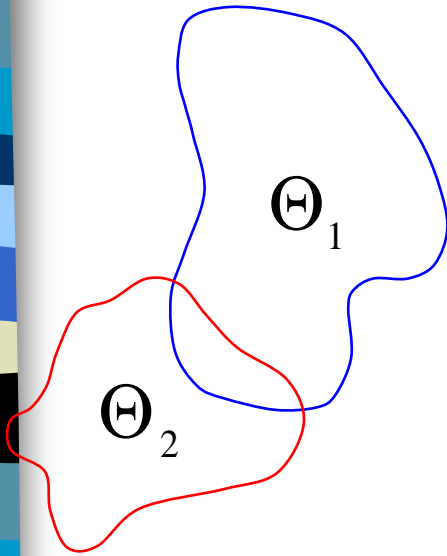
What are those models ?

Many possibilities but we'll talk only
of...

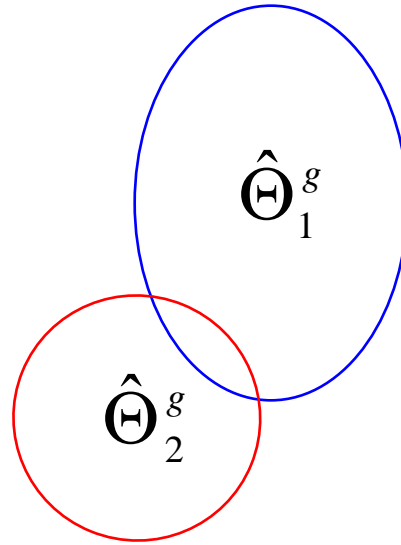
What are those models ?

... Hidden Markov Models with
Gaussian Distributions

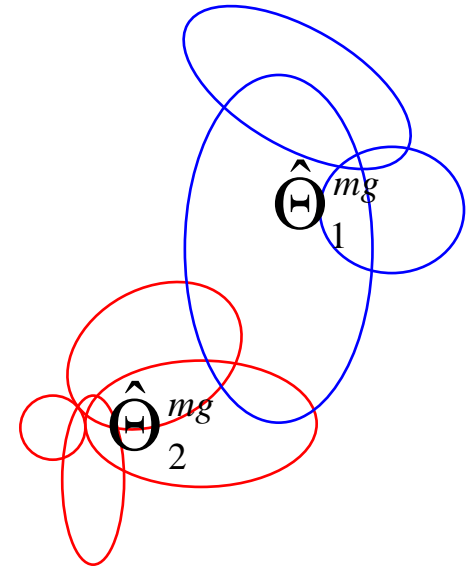
Gaussians



Real distribution



Gaussian model



Gaussian mixture model
(GMM)

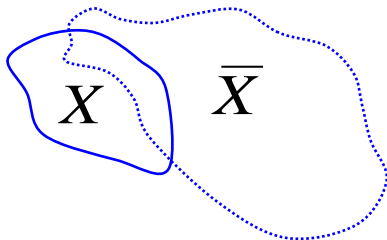


Automata

- For sequence processing
- Complex sequential patterns decomposed into piecewise stationary segments
- Each segment : deterministic or stochastic function
- Can describe grammar, lexicon, phone models...
- Example : Hidden Markov Models (HMMs)
 - 2 concurrent stochastic processes :
 - Sequence of HMM states (sequential structure of the data)
 - State output processes (local characteristics of the data)
 - Example : left-right HMM phone model with gaussian mixture output distributions

Different problems

Detection



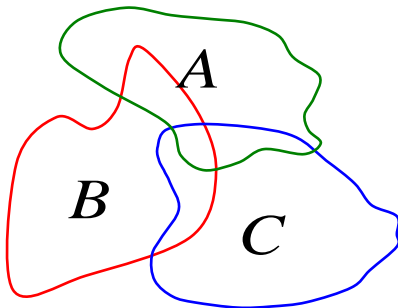
→ Binary decision tests

Segmentation



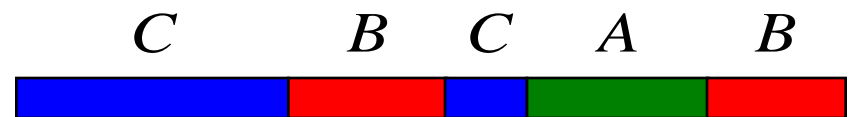
→ Change point detection

Clustering



→ Maximum A Posteriori

Decoding



→ State sequence search



Hidden Markov Models (HMMs)

Intro to HMMs

http://www.comp.leeds.ac.uk/roger/HiddenMarkovModels/html_dev/main.html



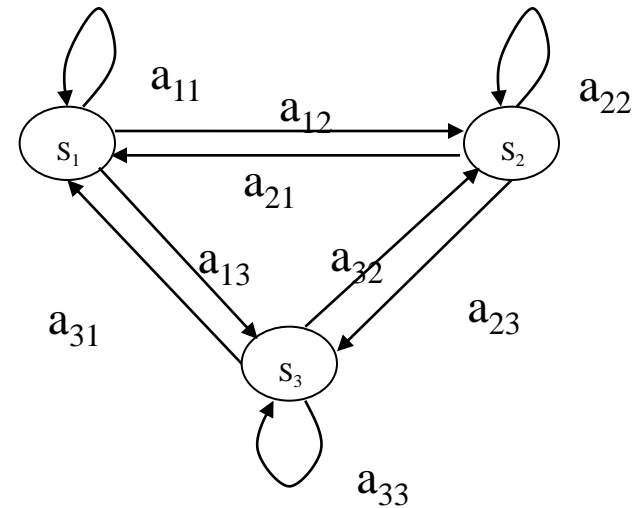
Hidden Markov Models (HMMs)

A HMM is defined by :

- N , number of states in the model, $S = \{S_1, S_2, \dots, S_N\}$
- M , number of output (emission) symbols per state, $V = \{v_1, v_2, \dots, v_M\}$
- Probability distributions are defined
 - Transition probabilities $A = \{a_{ij}\}$.
 - Emission probability of symbol k in state j : b_{jk}
 - Initial state probabilities $\pi = \{\pi_i\} \quad 1 \leq i \leq N$.

■ If the set of emission symbols V is finite, the HMM is called **discrete** (if V is infinite, then the HMM is continuous).

HMM for speech recognition



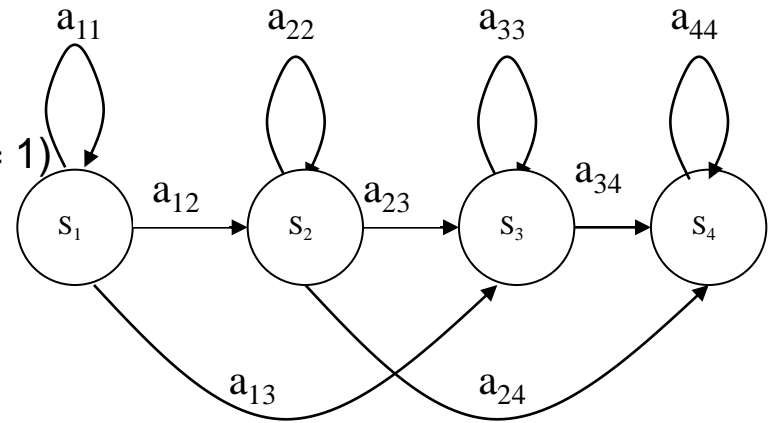
Ergodic HMM

Temporal aspect of speech

- Use of left-right HMMs (Bakis model).

Left-right HMM properties

- $a_{ij} = 0$ when $j < i$
- $(\pi_i = 0$ when $i \neq 1$) and $(\pi_1 = 1$ when $i = 1)$
- $a_{NN} = 1$



Left-right HMM



Three fundamental problems of HMMs

- Given observations O and HMM λ
 - How to calculate $P(O|\lambda)$?
 - The solution to this problem called **evaluation** is the algorithm *Forward-Pass*
- Given observations O and HMM λ
 - How to choose the most probable state sequence Q that maximizes $P(Q|O, \lambda)$?
 - The solution to this problem called decoding is the algorithm *Viterbi*
- Given observations O and HMM λ
 - How to adjust (train) the parameters of the model to maximize $P(O|\lambda)$? This is the **training** of the model parameters.
 - Algorithm Baum-Welch, algorithm **EM** (expectation-maximization)



Algorithm Forward pass (1)

- Sequence of T observations :

$$Y^{(k)} = y_{k_1}, \dots, y_{k_T}$$

- Partial probabilities (α 's) are calculated iteratively

$$\alpha_1(j) = \pi(j) \cdot b_{jk_1}$$

Algorithm Forward pass (2)

- Then for $t=2 \dots T$, :
$$\alpha_{t+1}(j) = \sum_{i=1}^n (\alpha_t(i) a_{ij}) b_{jk_t}$$
- Corresponding to the sum of the probabilities of each path leading to the considered state (j) multiplied by the emission probability in the considered state
- Finally, the sum of all the partial probabilities at time T, gives the probability of the observation given the HMM model

$$Pr(Y^{(k)}) = \sum_{j=1}^n \alpha_T(j)$$



Algorithm Viterbi (1)

- Calculate most probable state sequence

$$\mathbf{X}_i = (X_{i_1}, X_{i_2}, \dots, X_{i_T})$$

- From T observations

$$Y^{(k)} = y_{k_1}, \dots, y_{k_T}$$

- Partial probabilities (δ 's) are calculated iteratively

$$\delta_1(i) = \pi(i)b_{ik_1}$$

Algorithm Viterbi (2)

- Then for $t=2 \dots T$ and $i=1 \dots n$, one calculate

$$\delta_t(i) = \max_j (\delta_{t-1}(j) a_{ji} b_{ik_t})$$

$$\phi_t(i) = \operatorname{argmax}_j (\delta_{t-1}(j) a_{ji})$$

- Then, $i_t = \operatorname{argmax}(\delta_T(i))$ corresponds to the most probable state at time $t=T$
- Finally, « *backtracking* » is necessary to calculate the most probable path

$$i_t = \phi_{t+1}(i_{t+1})$$

FIN

