# Computing exactly with unsafe resources: fault tolerant exact linear algebra and cloud computing

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joint work with
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# Outline

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Inroduction
Distributed and Cloud computing
Fault tolerance
Parallel Exact linear algebra
Chinese Remainder and Error correcting code
Principle
Correction
Overview of the computation scheme
Further improvements
Ordering, sign
Early termination
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# High performance mathematics computing

Numerous application areas

**Number theory:**
- computing tables of elliptic curves, modular forms,
- testing of conjectures,

**Crypto:**
- Algebraic attacks
- Search for big primes

**Graph theory:** testing conjectures (graph isomorph.,...)

**Your research pb here!** ....

# Parallel computations

## (Re-)birth of parallel computing

Personal computers ⇒ multi/many cores

- End of the race for CPU clock-frequency
- Multi-core: 2, 4, 6, ...
- Many-cores: near future, already in GPU's
- multi-GPU

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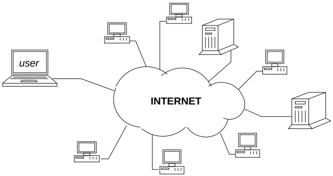
HPC ⇒Global computing

- servers
- grids, clusters,
- volunteer computing, Peer to Peer
- cloud computing




⇒Shift towards massively parallel computations

# Volunteer and Peer to peer computing

Well known successful projects:

- Mersenne Prime search
- SETI@Home
- Folding@Home/BOINC :
  ⇒1 Petaflops (670 000 PS3s) in 2007
- ...

⇒What level of trust?

# Cloud Computing

[Above the clouds, a Berkeley view on Cloud Computing, Feb 09]

Difficult agreement on the definition

Based on

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*The interesting thing about Cloud Computing is that we've redefined Cloud Computing to include everything that we already do... I don't understand what we would do differently in the light of Cloud Computing other than change the wording of some of our ads.*

*Larry Ellison*

*It's stupidity, It's worse than stupidity: it's a marketing hype campaign.*

*R. Stallman*

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Interest:

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$\Rightarrow$for massively parallel computations:
New *cost associativity* :

100's of computers for 1h $\equiv$ 1 computer for 100h

$\Rightarrow$What level of trust

# Fault tolerance

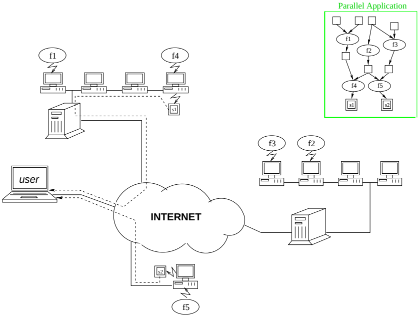
Need to deal with faults of different types

- Fail stop (crash failures)
- Network congestion
- Malicious attack

⇒Model of the Byzantine failure (not consistently failing)

# Global computing

- Grid and P2P: Transparent allocation of the resources to authenticated users
  *scheduling supports ressource connections / disconnections*



# Global computing and task forgery

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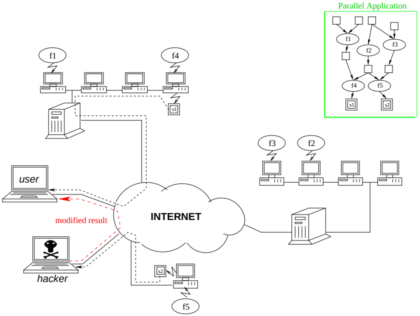


- Yet a task can be **forged** $\iff f(\text{ input }) \neq \hat{f}(\text{ input })$
- forgery can affect *many* tasks [e.g. patched client in SETI@home]

# State-of-the-Art in Result Certification

- Mainly target programs **P** composed of independent tasks
- General approach: *Replication-based*
  - Voting [*e.g.* BOINC, SETI@home]
  - Spot-checking [Germain-Playez03, based on Wald test]
  - checkpoint and restart
    $\Rightarrow$30' to 1h on large clusters!
  - Blacklisting, Credibility-based fault-tolerance [Sarmenta03]
  - Partial execution on reliable resources [Gao-Malewicz04]

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  - Partial execution on reliable resources [Gao-Malewicz04]
- Specific approach: *check post-condition* on the results
  - Eg: Sorting $\mathcal{O}(n \log n)$ – Simple-Checker $\mathcal{O}(n)$ [Blum97]
  - **The most efficient approach when possible!**

Yet, no guarantee of result correction without hypothesis on the attack

# Approach: Massive versus localized attacks

- In practice:
  - DAG of tasks, highly parallel
  - for most executions, only few or none forged tasks!
    $\hookrightarrow$ full replication useless and too expensive

- Yet, **no blind trust**:
  - large scale falsifications are possible $\hookrightarrow$ can be detected by trustable verification of randomly selected tasks:
    Extended Monte-Carlo Certification **EMCT**
    [Krings&al 06]
  - few falsifications are possible $\hookrightarrow$ can be efficiently corrected by Algorithm-based fault tolerance (**ABFT**)
    [Beckman 93, Plank&al 97, Saha 2006]

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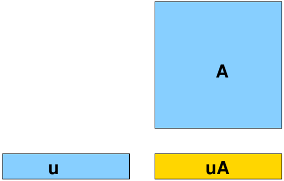

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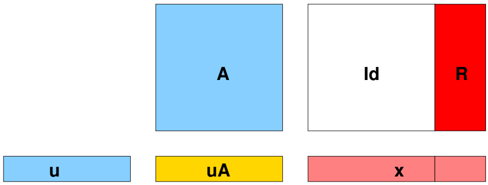

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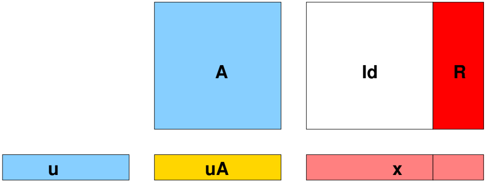


- precompute the product $B = A \times \begin{bmatrix} I & R \end{bmatrix}$
- compute $x = uB$ in parallel
- decode/correct $x$

# Parallel Exact linear algebra

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- Krylov iteration
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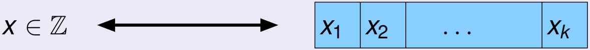
# Principle

## Chinese Remainder Theorem




where $p_1 \times \cdots \times p_k > x$ and $x_i = x \mod p_i \; \forall i$

# Principle

## Chinese Remainder Theorem

$x \in \mathbb{Z} \longleftrightarrow$

| | | | | | | |
|---|---|---|---|---|---|---|
| $x_1$ | $x_2$ | $\dots$ | $x_k$ | $x_{k+1}$ | $\dots$ | $x_n$ |

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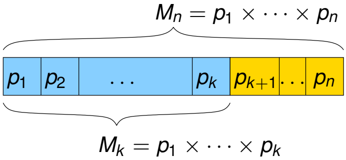
## Definition

*$(n,k)$-code:*
$C = \{(x_1, \ldots, x_n) \in \mathbb{Z}_{p_1} \times \cdots \times \mathbb{Z}_{p_n}$ *s.t. there exists* $x, 0 \leq x < p_1 \ldots p_k$ *and* $x_i = x \mod p_i \; \forall i\}$.

# Principle

## Property

$$X \in C \text{ iff } X < M_k.$$



# Principle

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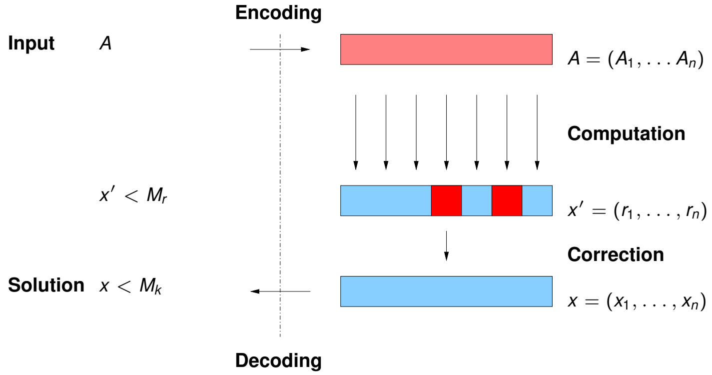
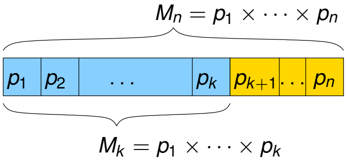


# Property of the code

Redundancy : $r = n - k$

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For $X' = X + E$ with $X \in C, \|E\| \leq r$,

$$X' > M_k.$$

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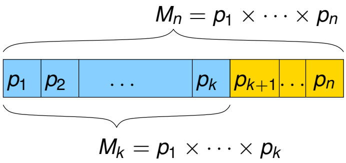


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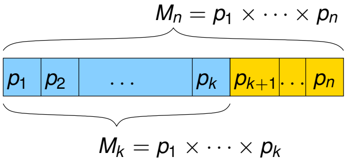


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# Correction

[Mandelbaum 78]

## CRT

- $\pi = p_1 \times \cdots \times p_n$,
- $\pi_i = \pi / p_i$
- $y_i = \pi_i^{-1} \mod p_i$.

$$x = \sum x_i \pi_i y_i$$

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[Mandelbaum 78]

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$$x = \sum x_i \pi_i y_i$$

- Error $e$ of weight $t \leq \left\lfloor \frac{r}{2} \right\rfloor$
- $C$: the product of the $p_i$ of the errors

$$\begin{aligned} x &= x' - e \\ &= x' - \sum_{\text{supp}(e)} x_i \pi_i y_i \\ &= x' - \frac{M_n}{C} B' \end{aligned}$$

for some $0 \leq B' \leq C$

# Correction

## Theorem

*If* $e \leq (n-k)\frac{\log p_{\min} - \log 2}{\log p_{\max} + \log p_{\min}}$

$$\left| \frac{x'}{M_n} - \frac{B'}{C} \right| \leq \frac{1}{C^2}$$

$\Rightarrow$ use continued fractions to approximate $\frac{x'}{M_n}$

$\Rightarrow$ recover $B'/C$ $\quad\Rightarrow e$

# Continued fractions

**Continued fractions:** Every rational can be written as

$$\frac{p}{q} = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}} = [a_0, \dots, a_n]$$

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$\Rightarrow$

- compute iteratively the convergents of $\frac{x'}{M_n}$ until $\left| \frac{x'}{M_n} - \frac{p_i}{q_i} \right| \leq \frac{1}{C^2}$ is satisfied.
- If $M_n \frac{p_i}{q_i} \in \mathbb{Z}$ and $x = x' - e \leq M_k$ $\Rightarrow$ Corrected

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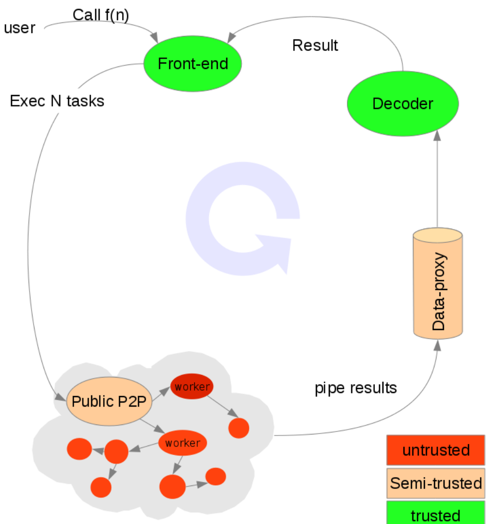
# Computation resources

3 types of resources:

- Trusted
  - 100% secure
- Untrusted
  - no security measures (public machines)
- Semi-Trusted
  - accessible by untrusted machines but with security measures
  - firewall, chroot, input checking...etc

# Basic scheme



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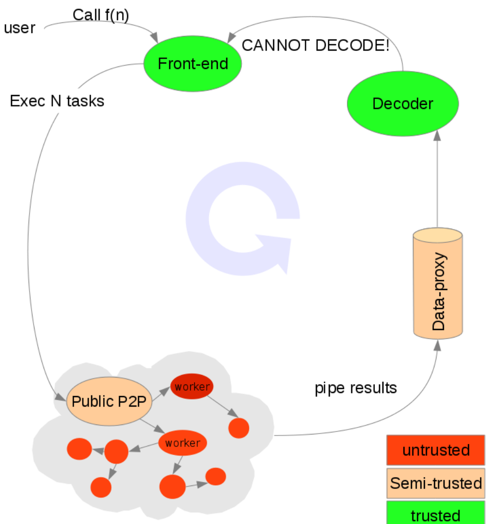
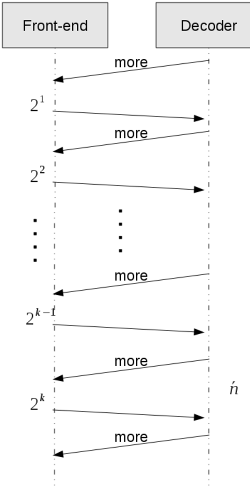


# Basic scheme

- we don't know the number of tasks!
- assuming the error rate is not fixed
- Trade-off: iterations vs #forks



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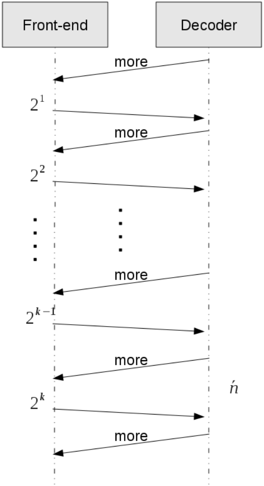
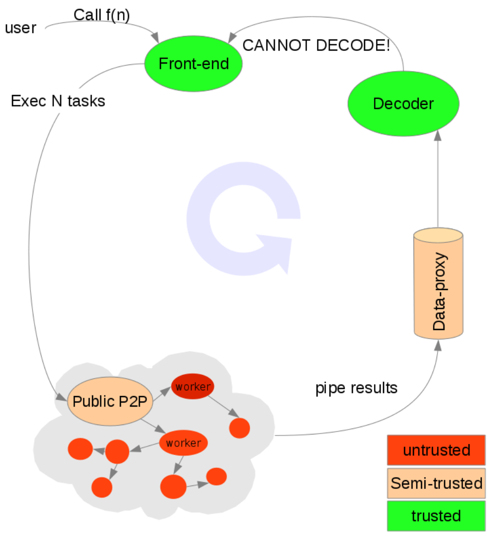



$\acute{n}$

increasing iteratively the number of tasks:

- quadratic growth: $2^i$
- $\rho^i$ with $1 < \rho < 2$
- $\rho$-amortized: $\rho^{f(i)}$ with $f$ sub-linear
  - $f(i) = i^\alpha$ with $0 < \alpha < 1$
  - $f(i) = \frac{i}{\log i}$, ....

# Online Post certification scheme



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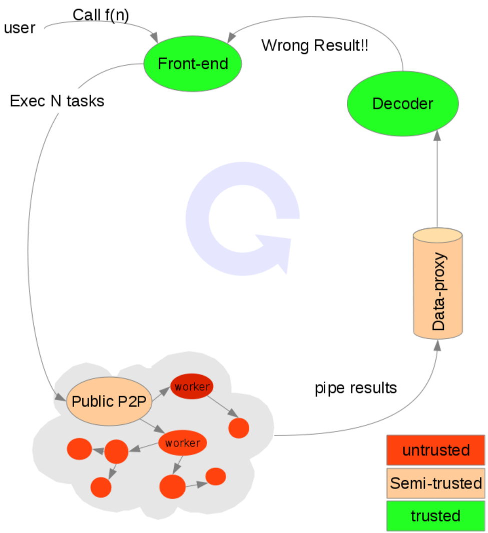

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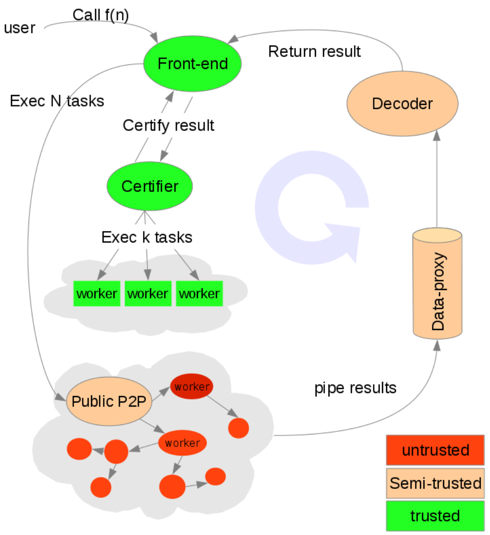
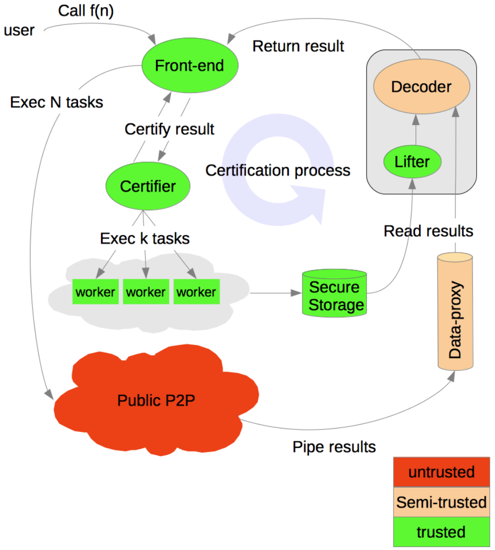


# Certification process

Read the decoder result $R'$
pick up a random prime $p_i$
compute $r = f$ over $\mathbb{Z}_{p_i}$
reduce $r' = R' \mod p_i$
**if** $r! = r'$ **then**
  Return ERROR.
**else**
  Return SUCCESS with a given probability
**end if**


Call f(n)
user
Front-end
Return result
Decoder
Exec N tasks
Certify result
Lifter
Certifier
Certification process
Read results
Exec k tasks
worker
worker
worker
Secure Storage
Data-proxy
Public P2P
Pipe results
untrusted
Semi-trusted
trusted

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# Removing the ordering of the moduli

worst case scenario :

every errors having a bigger contribution *at the end*,

Mandelbaum algorithm requires to sort the moduli $p_i$:

- to ensures accurately the $t = (n - k)/2$ correction rate
- to compute the optimal bound $C_{\max} = \prod_{i=n-k+1}^{n} p_i$ for the search of convergents.

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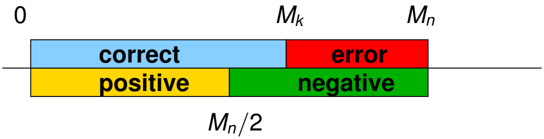
Relaxation:

- no sort, only find $p_{\min}$ and $p_{\max}$,
- more suited for a future *on-the-fly* model
- only an approximation of the distance (off by 1 in the worst cases)
- less accurate $C_{\max}$,

# Dealing with negative numbers

$$x \in C \text{ iff } 0 \leq x < M_k$$

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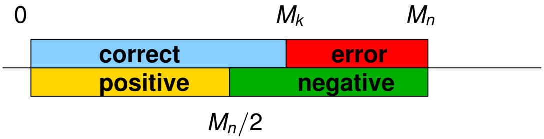


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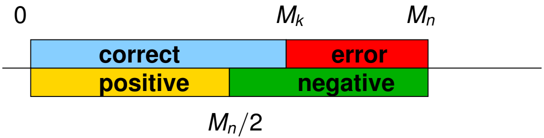


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Work-around solution:

- reconstruct both

$$x = CRT(x_1, \ldots, x_n)$$
$$y = CRT(-x_1, \ldots, -x_n)$$

- discard the bad one with the certifier

# Early termination

Chinese Remainder Algorithm requires a bound.
For Determinant ⇒Hadamard:

$$|\det(A)| \leq n^n (\max |a_{i,j}|)^{n/2} \leq \prod_i p_i$$

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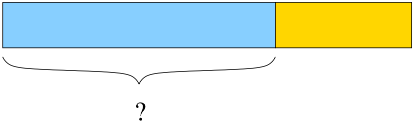
⇒**discover the bound during the computation**

- for each new modular result, reconstruct the result
- if it stabilizes $k$ times ⇒correct result with probability $1 - 1/p^k$

# Early termination

**Problem:** how to detect errors without a bound?



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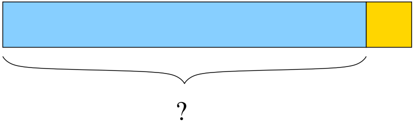
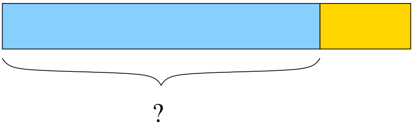



Multiple solution candidates:

- Solution $x_1$ with 1 error corrected

# Early termination

**Problem:** how to detect errors without a bound?




Multiple solution candidates:

- Solution $x_1$ with 1 error corrected
- Solution $x_2$ with 3 error corrected

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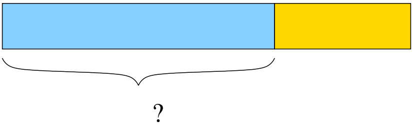


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- Solution $x_1$ with 1 error corrected
- Solution $x_2$ with 3 error corrected
- Solution $x_3$,with 2 errors corrected

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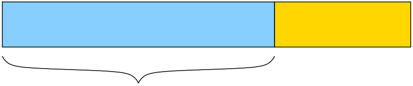
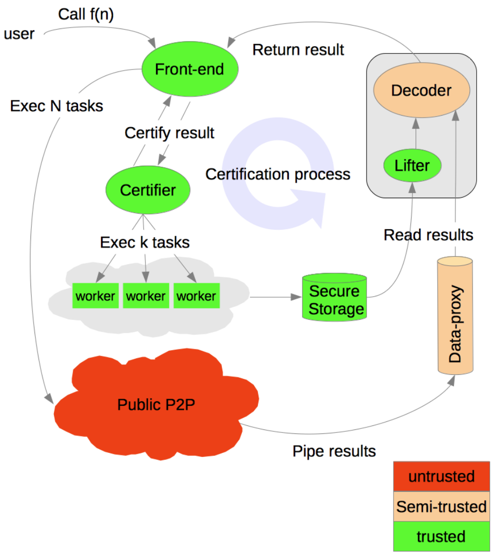



Multiple solution candidates:

- Solution $x_1$ with 1 error corrected
- Solution $x_2$ with 3 error corrected
- Solution $x_3$,with 2 errors corrected

The certifier discriminates the candidates

# Computation scheme



# Multiple decoding

**sign:** 2 candidates

**early term.:** multiple candidates, for each possible bound

Decoder returns a $l$ of candidates
**while** $|l| > 1$ **do**
  pick a random prime $p_i$
  Certifier compute $f_i = f \mod p_i$
  sieve un-compatible elements in $l$
  **if** $|l| = 1$ **then**
    return l.front()
  **end if**
**end while**

# Outline

# Parallelism : `Kaapi`

**Kernel for Adaptive Asynchronous Parallel Interface**

Virtualization of the parallel platform

- SMP, distributed architectures
- virtualizes the processors
  $\Rightarrow$programmer only focus on a high level description of the parallelism
- Scheduling:
  - static optimization from the data flow graph
  - dynamic using work stealing at fine grain granularity

# Parallelism : Kaapi
## Example of code

```
struct Fibonacci {
  void operator()( int n,              int& result )
    {
        if (n < 2) result =      n  ;
        else {
                       int  subresult1;
                       int  subresult2;
                     Fibonacci ()(n-1, subresult1);
                     Fibonacci ()(n-2, subresult2);
                     Sum ()(result, subresult1, subresult2);
        }
    }
};

struct Sum {
  void operator()(             int& result,
                          int  sr1,
                          int  sr2 )
    { result =      sr1        + sr2         ; }
}
```

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# Parallelism : Kaapi

## Example of code

```
struct Fibonacci {
  void operator()( int n, a1::Shared_w<int> result )
    {
        if (n < 2) result.write( n );
        else {
            a1::Shared<int> subresult1;
            a1::Shared<int> subresult2;
            a1::Fork<Fibonacci>()(n-1, subresult1);
            a1::Fork<Fibonacci>()(n-2, subresult2);
            a1::Fork<Sum>()(result, subresult1, subresult2);
        }
    }
};

struct Sum {
  void operator()( a1::Shared_w<int> result,
                   a1::Shared_w<int> sr1,
                   a1::Shared_w<int> sr2 )
    { result.write( sr1.read() + sr2.read() ); }
}
```

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# Computations

## Linear Algebra tasks: `LinBox`

- Fast MatMul (BLAS, delayed modular reductions, Strassen,...)
- Efficient, in-place, block Gaussian elimination

# Computations

## Linear Algebra tasks: `LinBox`

- Fast MatMul (BLAS, delayed modular reductions, Strassen,...)
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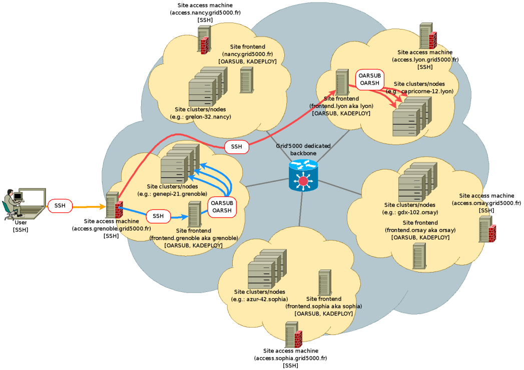
## Decoder: `Sage`

- ease of use for experimenting different prototypes
- good efficiency
- but BIG executable $\Rightarrow$hard to distribute

# The Grid 5000 architecture



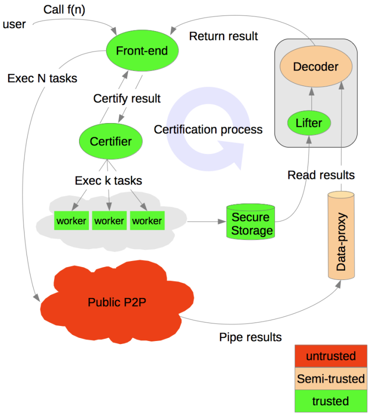

- Country-wide grid of 9 sites (Nancy, Lyon, Grenoble, Toulouse, Lille, Bordeaux, Orsay, Rennes, Sophia)
- Each site has 2-3 clusters of 60-600 computers
- heterogeneous architecture
  $\Rightarrow$ 3202 processors, 5714 cores

# Live demo

# Outline

# Perspectives

## A fully on-the-fly computation model:




- Need a fast CRT working *on the fly*
- Refine the amortized technique

# Sage ?

Revive the Dsage efforts?

distributed computations: Kaapi optional package?

- resource oblivious
- task parallelism, work-stealing, ABFT
- perspectives for GPU/multi-CPU support

P2P, volunteer computing

*Spare your cycles and contribute solving BSD conjecture!*