Network Economics
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Lecture 1: Pricing of communication services

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Fall 2016

References

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1. Introduction
2. The effect of congestion
3. Time dependent pricing
   – Parenthesis on congestion games and potential games
4. Pricing of differentiated services

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Examples of data pricing practices

- Residential Internet access
  – Most forfeits are unlimited
- Mobile data plans
  – AT&T moved to usage-based pricing in 2010
    - $10/GB
    - Stopped all unlimited plans in 2012
  – Verizon did the same
  – In France: forfeits with caps (e.g., 3GB for Free)

Why were there unlimited plans before?

- (Unlimited plans called flat-rate pricing)

- Users prefer flat-rate pricing
  - Willing to pay more
  - Better to increase market share
  - http://people.ischool.berkeley.edu/~hal/Papers/brookings/brookings.html
- The decrease in the cost of provisioning capacity exceeded the increase in demand

Why are providers moving to usage-based pricing?

- Demand is now growing faster than the amount of capacity per $
- Distribution of capacity demand is heavy-tailed: a few heavy users account for a lot of the aggregate

How to balance revenue and cost?

- Usage-based pricing
- Increase flat-rate price
  - Fairness issue
- Put a cap
- Slow down certain traffic or price higher premium service
  - See last section
  - Orange has a forfeit for 1000 Euros / month, all unlimited with many services. Their customers (about 1000 in France) got “macarons” to apologize for the disruption in 2012.

Generalities on setting prices

- Tariff: function which determine the charge r(x) as a function of the quantity x bought
  - Linear tariff: r(x) = p x
  - Nonlinear tariff
- Price design is an art, depends on the context
- 3 rationales
  - The price should be market-clearing
  - Competition, regulations (e.g., no cross-subsidization)
  - Incentive compatibility

Regulations

- Prices are often regulated by governments
  - Telecom regulators ARCEP (France), FCC (USA)
  - ≈ optimize social welfare (population + provider)
- Network neutrality debate
  - User choice
  - No monopoly
  - No discrimination
    - Provider-owned services
    - Protocol-level
    - Differentiation of consumers by their behavior
    - Traffic management and QoS
- Impact on peering economics

Modeling: consumer problem

- Set of consumers N = {1, …, n}
- Each consumer chooses the amount x consumed to maximize his utility – cost
- Under linear tariff (usage-based price p)

$$x_i(p) = \arg\max_x [u_i(x) - px]$$

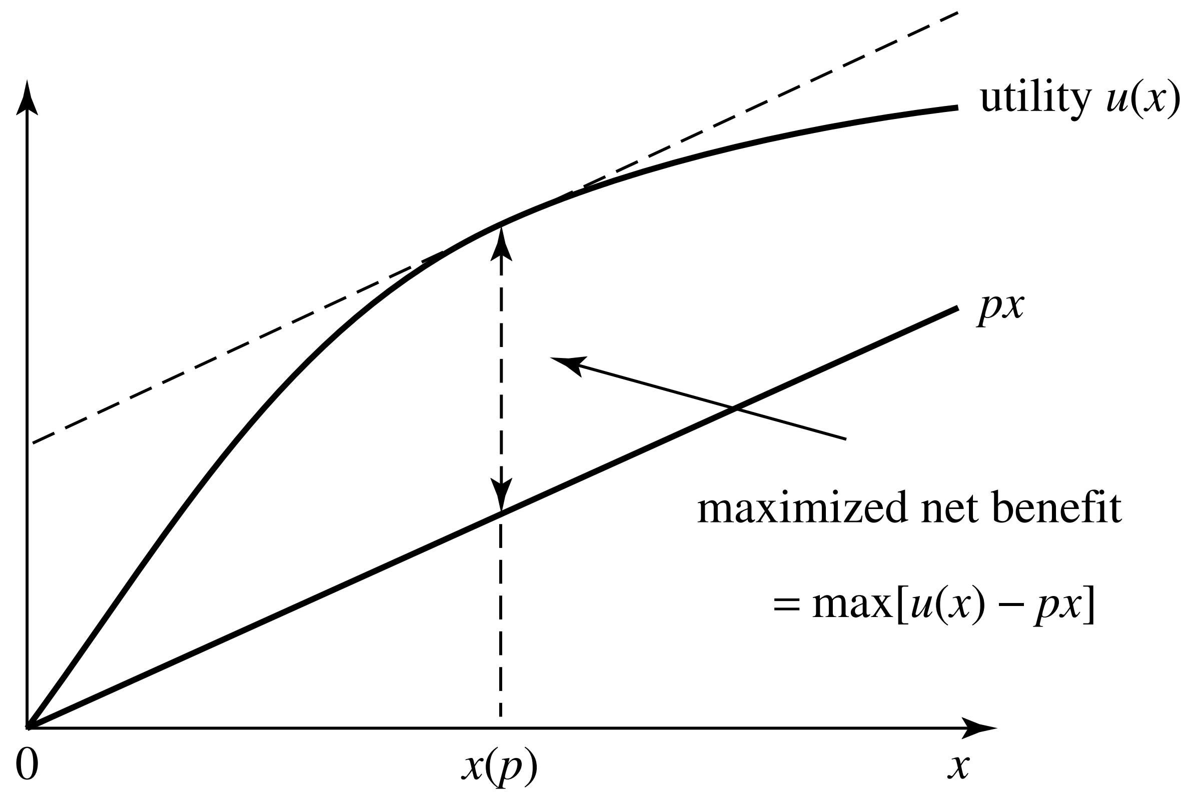
- Consumer surplus

$$CS_i = \max_x [u_i(x) - px]$$

- u(x) assumed concave

Consumer utility

- Example: u(x) = log(x) (proportional fairness)



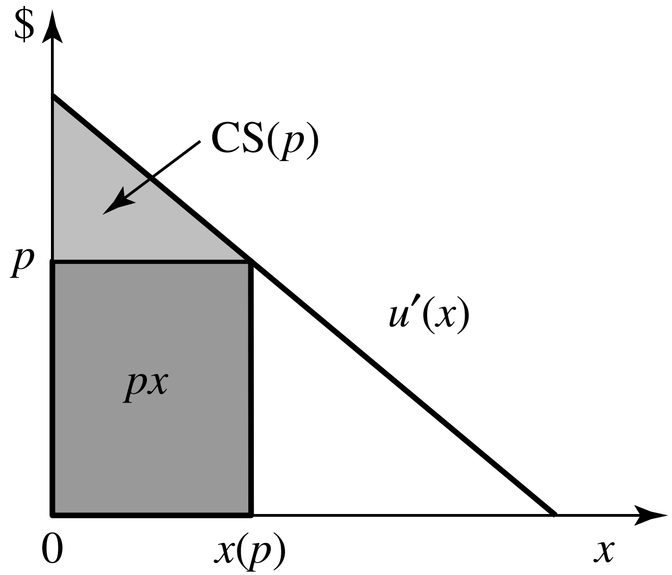
Demand functions

- Individual demand $x_i(p) = (u_i')^{-1}(p)$
- Aggregate demand $D(p) = \sum_{i \in N} x_i(p)$

- Inverse demand function: p(D) is the price at which the aggregate demand is D
- For a single customer: $p(x) = u'(x)$

Illustrations

- Single user $CS(p) = \int_0^{x(p)} p(x)dx - px$



- Multiple users: replace u’(x) by p(D)

Elasticity

- Definition: $$\varepsilon = \frac{\partial D(p) / \partial p}{D(p) / p}$$
- Consequence: $$\frac{\Delta D}{D} = \varepsilon \frac{\Delta p}{p}$$
- $|\varepsilon|>1$: elastic
- $|\varepsilon|<1$: inelastic

Provider’s problem: choose a tariff

- Many different tariffs

- Choosing the right one depends on context (art)
  - User demand; costs structure; regulation; competition
- More information:
  - R. Wilson. “Nonlinear pricing”, OUP 1997.

Flat-rate vs usage-based pricing

- Flat-rate: equivalent to $p=0$
  - There is a subscription price, but it does not play any role in the consumer maximization problem
- Illustration

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The problem of congestion

- Until now, we have not seen any game
- One specificity with networks: congestion (the more users the lower the quality)
  - Externality
- Leads to a tragedy of the commons

Tragedy of the commons (1968)

- Hardin (1968)
- Herdsmen share a pasture
- If a herdsman add one more cow, he gets the whole benefit, but the cost (additional grazing) is shared by all
- Inevitably, herdsmen add too many cows, leading to overgrazing

Simple model of congestion

- Set of users N = {1, …, n}
- Each user i chooses its consumption $x_i \geq 0$
- User i has utility

$$u_i(x) = f(x_i) - (x_1 + \ldots + x_n)$$

  - f(.) twice continuously differentiable increasing strictly concave
- We have a game! (one-shot)

Simple model: Nash equilibrium and social optimum

- NE: user i chooses x_i such that

$$f'(x_i) - 1 = 0$$

- SO: maximize

$$\sum_{i \in N} u_i(x) = \sum_{i \in N} [f_i(x) - (x_1 + \ldots + x_n)]$$

→Gives for all i: $f'(x_i) - n = 0$

- Summary: $x_i^{NE} = f'^{-1}(1)$

$$x_i^{SO} = f'^{-1}(n)$$

Illustration

Price of Anarchy

- Definition: $$PoA = \frac{\text{Welfare at SO}}{\text{Welfare at NE}}$$

- If several NE: worse one
- Congestion model: $$PoA = \frac{f(x^{SO}) - nx^{SO}}{f(x^{NE}) - nx^{NE}}$$

- Unbounded: for a given n, we can find f(.) such that PoA is as large as we want
- Users over-consume at NE because they do no fully pay the cost they impose on others

Congestion pricing

- One solution: make users pay the externality on the others, here user i will pay (n-1) x_i
- Utility becomes

$$u_i(x) = f(x_i) - (x_1 + \ldots + x_n) - (n-1)x_i$$

- FOC of NE is the same as SO condition, hence selfish users will choose a socially optimal consumption level
- We say that the congestion price "internalizes the externality"

Pigovian tax and VCG mechanism

- A. Pigou. “The Economics of Welfare” (1932).
  - To enforce a socially optimal equilibrium, impose a tax equal to the marginal cost on society at SO

- Vickrey–Clarke–Groves mechanism (1961, 1971, 1973): a more general version where the price depends on the actions of others
  - See later in the auctions lecture

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Different data pricing mechanisms ("smart data pricing")

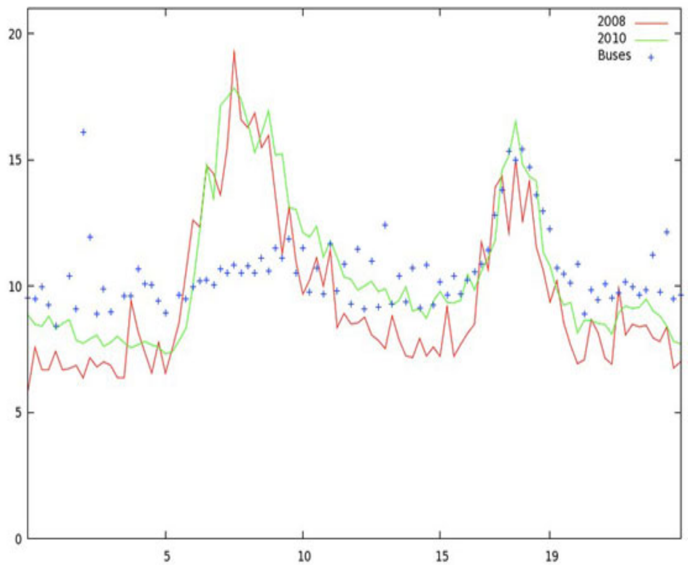
- Priority pricing (SingTel, Singapore)
- Two-sided pricing (Telus, Canada; TDC, Denmark)
- Location dependent pricing (in transportation networks)
- Time-dependent pricing
  – Static
  – Dynamic

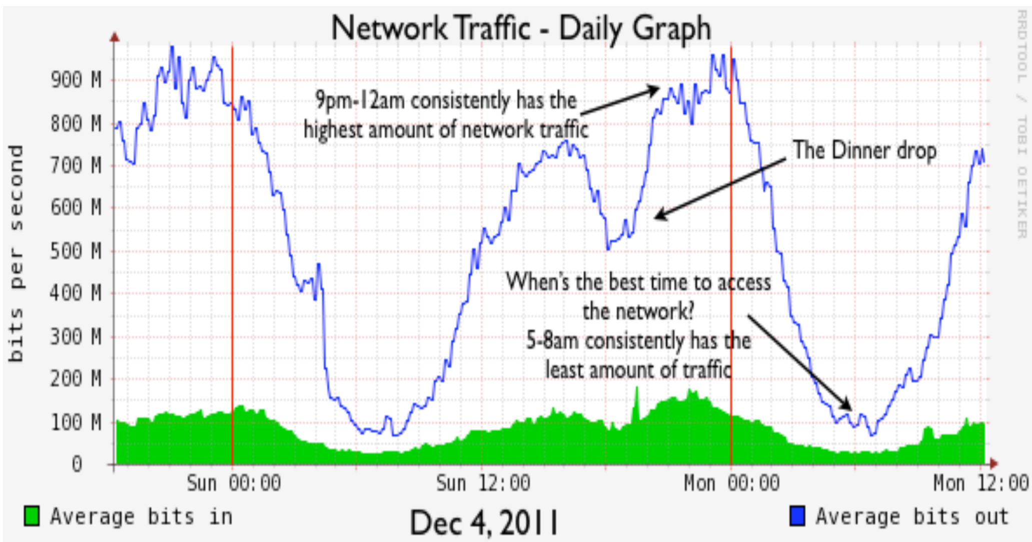
Examples

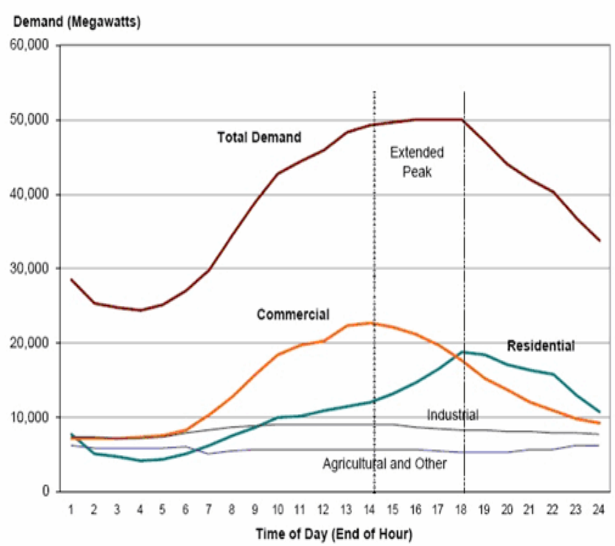
- Orange UK has a “happy hours” plan
  - Unlimited during periods: 8-9am, 12-1pm, 4-5pm, 10-11pm
- African operator MTN uses dynamic tariffing updated every hour
  - Customers wait for cheaper tariffs
- Unior in India uses congestion dependent pricing

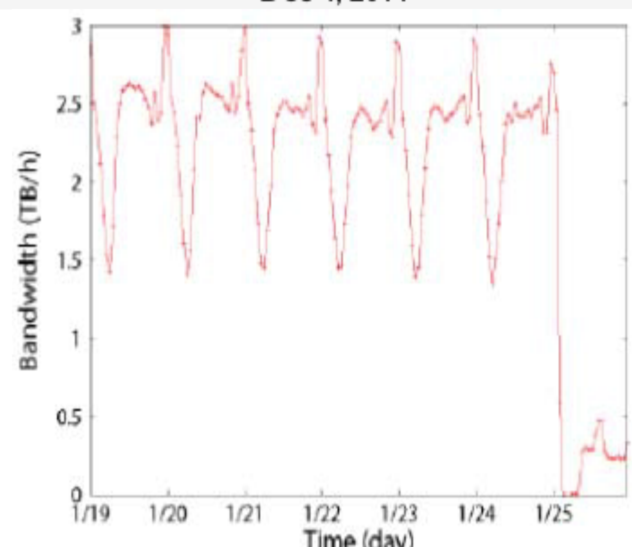
Different applications

Daily traffic pattern









Models of time-dependent pricing

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  - Waiting function
  - Implementation (app)
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- P. Loiseau, G. Schwartz, J. Musacchio, S. Amin and S. S. Sastry. "Incentive Mechanisms for Internet Congestion Management: Fixed-Budget Rebate versus Time-of-Day Pricing", IEEE/ACM Transactions on Networking, 2013 (to appear).

Model

- T+1 time periods {0, …, T}
  - 0: not use the network
- Each user
  - class c in some set of classes
  - chooses a time slot to put his unit of traffic
  - x_t^c: traffic from class c users in time slot t ($x^c = \sum_t x_t^c$)
- Large population: each user is a negligible fraction of the traffic in each time slot
- Utility of class c users: $u_c = u_0 - \left[g_t^c + d(N_t) 1_{t>0} \right]$
  - g_t^c: disutility in time slot t
  - N_t: traffic in time slot t ($N_t = \sum_c x_t^c$)
  - d(.): delay – increasing convex function

Equivalence with routing game

- See each time slot as a separate route
- Rq: each route could have a different delay

Wardrop equilibrium (1952)

- Similar to Nash equilibrium when users have negligible contribution to the total
  - A user's choice does not affect the aggregate
  - Called non-atomic
- Wardrop equilibrium: a user of class c is indifferent between the different time slots (for all c)
  - Implies that all time slots have the same disutility for each class: there exists λ_c's such that

$$g_t^c + d(N_t)1_{t>0} = \lambda_c, \quad \text{for all } t \text{ and all } c$$

Example

- 1 class, $g_1=1$, $g_2=2$, $d(N)=N^2$, $N_1+N_2=2$

Social optimum

- Individual utility for class c users

$$u_c = u_0 - \left[g_t^c + d(N_t) 1_{t>0} \right]$$

- Social welfare:

$$W = Nu_0 - \sum_t \left[\sum_c \left[x_t^c g_t^c \right] + N_t d(N_t) 1_{t>0} \right]$$

- How to achieve SO at equilibrium?

$$u_c = u_0 - \left[g_t^c + d(N_t) 1_{t>0} + p_t \right]$$

  – p_t: price in time slot t

Achieving SO at equilibrium

- Theorem: If

$$p_t = N_t d'\left(N_t\right)$$

  then the equilibrium coincides with SO.
- This price internalizes the externality

Proof

(Congestion games)

- Previous example: each user chooses a resource and the utility depends on the number of users choosing the same resource
- Particular case of congestion games
  - Set of users {1, …, N}
  - Set of resources A
  - Each user i chooses a subset $a_i \subset A$
  - n_j: number of users of resource j ($n_j = \sum_{i=1}^{N} 1_{j \in a_i}$)
  - Utility: $u_i = -\sum_{j \in a_i} g_j(n_j)$
    - g_j increasing convex

(Potential games: definition)

- Game defined by
  - Set of users N
  - Action spaces A_i for user i in N
  - Utilities $u_i(a_i, a_{-i})$
- … is a potential game if there exists a function Φ (called potential function) such that

$$u_i(a_i, a_{-i}) - u_i(a_i', a_{-i}) = \Phi(a_i, a_{-i}) - \Phi(a_i', a_{-i})$$

- i.e., if i changes from a_i to a_i', his utility gain matches the potential increase

(Potential games examples)

- Battle of the sexes

P1 \ P2	alpha	beta
alpha	2, 1	0, 0
beta	0, 0	1, 2

(Potential games examples 2)

- Battle of the sexes more complex

		P2 alpha	P2 beta
P1	alpha	5, 2	-1, -2
P1	beta	-5, -4	1, 4

(Potential games examples 3)

- Heads and tails

P2

		heads	tails
P1	heads	1, -1	-1, 1
	tails	-1, 1	1, -1

(Properties of potential games)

- Theorem: every finite potential game has at least one pure strategy Nash equilibrium (the vector of actions maximizing Φ)
- More generally: the set of pure strategy Nash equilibria coincides with the set of local maxima of the potential Φ
- Many other properties on PoA, etc.

(Properties of potential games 2)

- Best-response dynamics: players sequentially update their action choosing best response to others actions
- Theorem: In any finite potential game, the best-response dynamics converges to a Nash equilibrium
- Useful for distribution optimization algorithm design
  - Channel selection/power allocation in wireless

(Congestion games vs potential games)

- Congestion games are potential games (Rosenthal 1973)
- Potential games are congestion games (Monderer and Shapley 1996)

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Paris Metro Pricing (PMP)

- One way to increase revenue: price differentiation

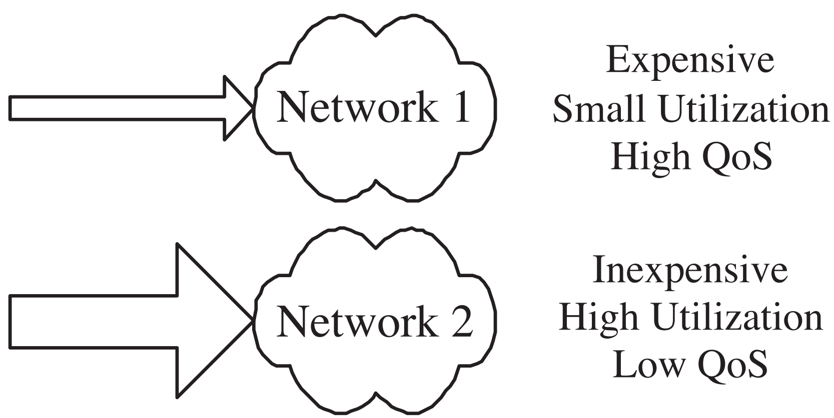
- PMP: Simplest possible type of differentiated services
- Differentiation is created by the different price
- Famous paper by A. Odlyzko in 1999
- Used in Paris metro in the 70’s-80’s

PMP toy example

- Network such that
  - Acceptable for VoIP if ≤ 200 users
  - Acceptable for web browsing if ≤ 800 users
- Demand
  - VoIP demand of 100 if price ≤ 20
  - Web browsing demand of 400 if price ≤ 5
- How to set the price?
  - Charge 20: revenue of 20x100 = 2,000
  - Charge 5: revenue of 5x400 = 2,000

PMP toy example (2)

- Divide network into 2 identical subnetwork
- Each acceptable
  - for VoIP if ≤ 100 users
  - for web browsing if ≤ 400 users
- Charge 5 for one, 20 for the other
  - Revenue 100x20 + 400x5 = 4,000



Population model

- N users
- Network of capacity 2N
- Each user characterized by type θ
- Large population with uniform θ in [0, 1]
- Each user finds network acceptable if the number of users X and price p are such that

$$\frac{X}{2N} \leq 1-\theta \quad \text{and} \quad p \leq \theta$$

Revenue maximization

- Assume price p
- If X users are present, a user of type θ connects if $\theta \in [p, 1 - X/2N]$
- Number of connecting users binomial with mean

$$N(1 - X/(2N) - p)^+$$

- So, $$\frac{X}{N} \approx \left(1 - \frac{X}{2N} - p\right)^+ \Rightarrow \frac{X}{N} = \frac{2-2p}{3}$$
- Maximizing price: p=1/2, revenue N/6

PMP again

- Divide the network in two, each of capacity N
- Prices are p_1 and p_2, acceptable if

$$\frac{X}{N} \leq 1-\theta \quad \text{and} \quad p_i \leq \theta$$

- If both networks are acceptable, a user takes the cheapest
- If both networks are acceptable and at the same price, choose the lowest utilization one
- Maximal revenue:
  - p_1=4/10, p_2=7/10
  - Revenue Nx9/40 → 35% increase

Competition

- What if the two sub-networks belong to two different operators?
- Maximum total revenue would be with
  - One at p_1=4/10 → revenue Nx12/100
  - One at p_2=7/10 → revenue Nx21/100
- But one provider could increase his revenue

Competition (2)

- There is no pure strategy NE