Network Economics
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Lecture 5: Auctions and applications

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References

- V. Krishna, “Auction Theory”, Elseiver AP 2009 (second edition)
  - Chapters 2, 3, 5
- P. Milgrom, “Putting auction theory to work”, CUP 2004
  - Chapter 1
- D. Easley and J. Kleinberg, “Networks, Crowds and Markets”, CUP 2010
  - Chapters 9 and 15
- Ben Polak’s online course http://oyc.yale.edu/economics/econ-159
  - Lecture 24

Outline

1. Generalities on auctions
2. Private value auctions
3. Common value auctions: the winner's curse
4. Mechanism design
5. Generalized second price auction

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Where are auctions?

- Everywhere!
  - Ebay
  - Google search auctions
  - Spectrum auctions
  - Art auctions
  - Etc.

What is an auction?

- Seller sells one item of good through bidding
  - Set of buyers
- Buyer buys one item of good
  - Set of sellers
  - Called procurement auction (governments)
- Auctions are useful when the valuation of bidders is unknown
- More complex auctions
  - Multi-items
  - Combinatorial

Standard auction

- Standard auction: the bidder with the highest bid wins
- Example of nonstandard auction: lottery

The two extreme settings

- Common values ←→ Private values

Main types of auctions

1. Ascending open auction (English)

2. Descending open auction (Dutch)

3. First-price sealed bid auction

4. Second price sealed bid auction (Vickrey)

Relationships between the different types of auctions

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Private value auctions: Model

- One object for sale
- N buyers
- Valuation X_i
- X_i's i.i.d. distributed on [0, w], cdf F(.)
- Bidder i knows
  - Realization x_i of his value
  - That other bidders have values distributed according to F
- Def: symmetric: all bidders have the same distribution of value

Game

- The game is determined by the auction rules
  - Game between the bidders
- Bidder's strategy: β_i: [0, w] → [0, ∞)

- Look for symmetric equilibria
  - 1st price auction
  - 2nd price auction
  - Compare seller's revenue

Second-price sealed-bid auction

- Proposition: In a second-price sealed-bid auction, bidding its true value is weakly dominant

First-price sealed-bid auction

- Bidding truthfully is weakly dominated

First-price sealed-bid auction (2)

- What is the equilibrium strategy?

First-price sealed-bid auction (3)

- Proposition: Symmetric equilibrium strategies in a first-price sealed-bid auction are given by

$$\beta(x) = E\left[Y_1 \mid Y_1 < x\right]$$

where Y_1 is the maximum of N-1 independent copies of X_i

Example

- Values uniformly distributed on [0, 1]

Revenue comparison

- With independently and identically distributed private values, the expected revenue in a first-price and in a second-price auction are the same

Proof

Warning

- This is not true for each realization
- Example: 2 bidders, uniform values in [0, 1]

Revenue equivalence theorem

- Generalization of the previous result

- Theorem: Suppose that values are independently and identically distributed and all bidders are risk neutral. Then any symmetric and increasing equilibrium of any standard auction such that the expected payment of a bidder with value zero is zero yields the same expected revenue to the seller.

- See an even more general result in the (beautiful) paper R. Myerson, “Optimal Auction Design”, Mathematics of Operation Research 1981
  - 2007 Nobel Prize

Proof

Reserve price

- $r>0$, such that the seller does not sell if the price determined by the auction is lower

Reserve price in second-price auction

- No bidder with value $x<r$ can make a positive profit
- Bidding truthfully is still weakly dominant
- Winner pays r if the determined price is lower
- Expected payment

Reserve price in first-price auction

- No bidder with value $x<r$ can make a positive profit
- Symmetric equilibrium:

- Expected payment:

Effect of reserve price on revenue

- Seller has valuation x_0 of the good
- Sets $r>x_0$!

- Optimal reserve price:

- Increases the seller’s revenue
  - Sometimes called exclusion principle

Remark

- Efficiency: maximize social welfare
  - Good ends up in the end of the highest value among bidders and seller
- Efficient is NOT the same as revenue optimality
- Example
  - Seller with valuation zero

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Playing with a jar of coins

The winner's curse

- Good has value V, same for all bidders
  - Example: oil field
- Each bidder has an i.i.d. estimate $x_i = V + e_i$ of the value ($E(e_i)=0$)
- They all bid (e.g., first-price auction)

The winner's curse (2)

- Suppose bidder 1 wins
- Upon winning, he finds out his estimate was too large! → bad news: winner's curse

- Bid as if you know you win!

- Remark: the winner's curse does not arise at equilibrium, if your bid takes it into account.

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Mechanism design

- An auction is only one of many ways to sell a good
- Mechanism design studies the design of rules such that the resulting game yields a desired outcome
- The 2007 Nobel Memorial Prize in Economic Sciences was awarded to Leonid Hurwicz, Eric Maskin, and Roger Myerson "for having laid the foundations of mechanism design theory"

Setting

- Buyers
- Values
- Set of values
- Distributions
- Product set
- Joint density

Mechanisms

- Set of messages (bids)
- Allocation rule
- Payment rule

- Example: 1st or 2nd price auction

Direct mechanism

- Definition
- Characterization: Pair (Q, M)
- Truthful equilibrium

Revelation principle

- Given a mechanism and an equilibrium for that mechanism, there exists a direct mechanism such that
  1. It is an equilibrium for each buyer to report his value truthfully
  2. The outcomes (probabilities Q and expected payment M) are the same as in the equilibrium of the original mechanism

Proof

Incentive compatibility (IC)

- A direct revelation mechanism is IC if it is optimal for a buyer to report his value truthfully when all other buyers report their value truthfully

Revenue equivalence

- If the direct mechanism (Q, M) is incentive compatible, then the expected payment is

$$m_i(x_i) = m_i(0) + q_i(x_i)x_i - \int_0^{x_i} q_i(t_i)dt_i$$

- Thus, the expected payment in any two incentive compatible mechanisms with the same allocation rule are equivalent up to a constant

- Generalizes the previous version

Two questions

- How to design a revenue optimal mechanism?
- How to design an efficient mechanism?
- Restricting to
  - IC mechanisms
  - Individually rational mechanisms (i.e., such that the expected payoff of every buyer is nonnegative)

Optimal mechanism

- Define the virtual valuation $\psi_i(x_i) = x_i - \frac{1 - F_i(x_i)}{f_i(x_i)}$

- Define $y_i(x_{-i}) = \inf\left\{z_i : \psi_i(z_i) \geq 0 \text{ and } \psi_i(z_i) \geq \psi_j(x_j) \text{ for all } j\right\}$

- Under some regularity conditions, the optimal mechanism is: allocate to the buyer with highest virtual valuation (if it is nonnegative), with expected payment $y_i(x_{-i})$

Symmetric case

- We find the second price auction with reserve price $\psi^{-1}(0)$

Efficient mechanism

- Social welfare maximized by Q^*
- If there is no tie: allocation to the buyer with highest value

- Notation:

VCG mechanism: definition

- The VCG mechanism is (Q^{*}, M^{V}), where

$$M_i^V(x) = W(0, x_{-i}) - W_{-i}(x)$$

- Note: the W's are computed with the efficient allocation rule

VCG mechanism: properties

- The VCG mechanism is
  - Incentive compatible – truthful reporting is weakly dominant
  - Individually rational
  - Efficient

- i's equilibrium payoff is the difference in social welfare induced by his truthful reporting instead of 0

- Proposition: Among all mechanisms for allocating a single good that are efficient, IC and IR, the VCG mechanism maximizes the expected payment of each agent

Example

- In the context of auctions: VCG = 2^{nd} price auction!

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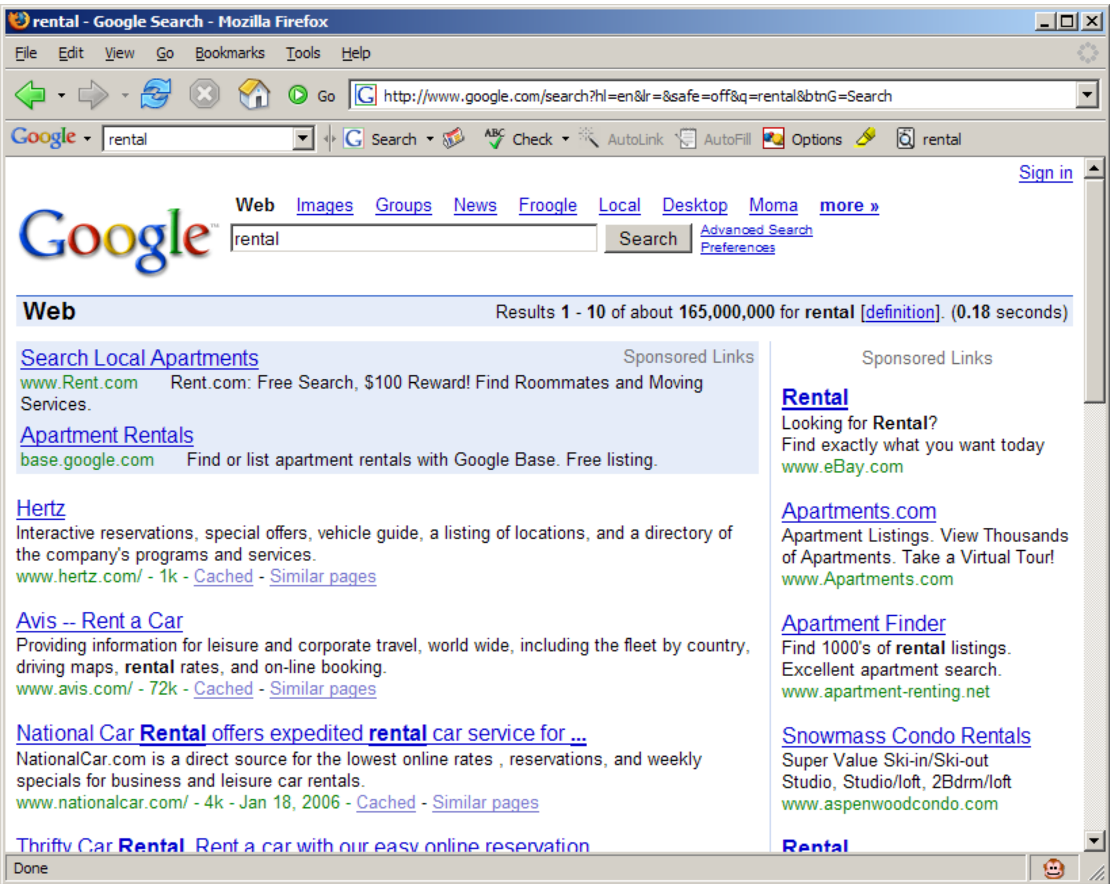
Sponsored search



- Ads in sponsored box
- Several spots: multiple items auction
- Pay per click for the advertiser

Generalized second price auction (GSP)

- How does Google determine which ad is shown for a given keyword?

- Advertisers submit bids
- Google ranks ads by bid x expected nb of clicks
  - Ad quality factor

- Advertisers pay the price determined by the bid below (GSP)

GSP properties

- GSP is not truthful
- GSP is not VCG
- GSP may have several equilibria
- GSP's revenue may be higher or lower than VCG's revenue

- B. Edelman, M. Ostrovsky, M. Schwarz, "Internet Advertising and the Generalized Second-Price Auction: Selling Billions of Dollars Worth of Keywords", American Economic Review 2007
- H. Varian, "Position auctions", International Journal of Industrial Organization 2007