Statistical Discrimination in Stable Matchings

RÉMI CASTERA, Univ. Grenoble Alpes, Inria, CNRS, Grenoble INP, LIG, France
PATRICK LOISEAU, Univ. Grenoble Alpes, Inria, CNRS, Grenoble INP, LIG, France
BARY S.R. PRADELSKI, Univ. Grenoble Alpes, Inria, CNRS, Grenoble INP, LIG, France

EXTENDED ABSTRACT

Statistical discrimination results when a decision-maker observes an imperfect estimate of the quality of each candidate dependent on which demographic group they belong to [1, 8]. Imperfect estimates have been modelled via noise, where the variance depends on the candidate's group ([4, 6, 7]). Prior literature, however, is limited to simple selection problems, where a single decision-maker tries to choose the best candidates among the applications they received.

In this paper, we initiate the study of statistical discrimination in matching, where multiple decision-makers are simultaneously facing selection problems from the same pool of candidates. We consider the college admission problem as first introduced in [5] and recently extended to a model with a continuum of students [3]. We propose a model where two colleges A and B observe noisy estimates of each candidate's quality, where W^s, the vector of estimates for student s, is assumed to be a bivariate normal random variable. In this setting, the estimation noise controls a new key feature of the problem, namely correlation, ρ, between the estimates of the two colleges: if the noise is high, the correlation is low and if the noise is low the correlation is high.

We assume that the population of students is divided into two groups G_1 and G_2, and that members of these two groups are subject to different correlation levels between their grades at colleges A and B. Concretely, for each student s, their grade vector (W_A^s, W_B^s) is drawn according to a centered bivariate normal distribution with variance 1 and covariance ρ_{G_s}, where G_s is the group student s belongs to. We consider the stable matching induced by this distribution and characterize how key outcome characteristics vary with the parameters, in particular with the group-dependent correlation coefficient. Our results summarize as follows:

(1) We show that the probability that a student is assigned to their first choice is independent of the student's group, but that it decreases when the correlation of either group decreases. This means that higher measurement noise (inducing lower correlation) on one group hurts not only the students of that group, but the students of all groups.
(2) We show that the probability that a student is assigned to their second choice and the probability that they remain unassigned both depend on the student's group, which reveals the presence of statistical discrimination coming from the correlation effect alone. Specifically, we find that the probability that a student remains unmatched is decreasing when the correlation of their group decreases (higher measurement noise) and when the correlation of the other group increases. In other words, the higher the measurement noise of their own group, the better off students are with regard to getting assigned a college at all. This is somewhat counter-intuitive, but is explained by the observation that with high noise (i.e., low correlation) the fact that a student is rejected from one college gives only little information about the outcome at the other college. That is, a student has an independent second chance for admission.

These two comparative static results give insights on the effect of correlation on the stable matching outcome for different demographic groups and show that indeed, statistical discrimination is an important

EC '22, July 11–15, 2022, Boulder, CO, USA.

ACM ISBN 978-1-4503-9150-4/22/07.
https://doi.org/10.1145/3490486.3538364

theory to understand discrimination in matching problems. We also analyze a number of special cases of our model, in particular the case of a single group, to show that even in this case correlation affects the outcome. It is interesting to notice that the effect of correlation on the number of students getting their first choice in our model is the same as in [2], i.e., a higher correlation leads to more students getting their first choice.

Our work is the first to investigate statistical discrimination in the context of matching. Overall we find that group-dependent measurement noises of the candidates quality—and the resulting group-dependent correlation between the colleges' estimates—play an important role in leading to unequal outcomes for different demographic groups, and in particular underrepresentation of one of the groups. Of course, we do not argue that statistical discrimination is the only possible cause of discrimination. In particular, if there is bias in the quality estimates for one group, then it will naturally also hurt the representation of that group. We do not model bias since our primary purpose is to isolate the effect of statistical discrimination. Throughout the paper, we make a number of other simplifying assumptions (e.g., focusing on two colleges) whose purpose is also to simplify our results and isolate the effect of correlation. Our analysis, however, extends to more general contexts.

The full paper is available at https://hal.archives-ouvertes.fr/hal-03672270.

CCS Concepts: • **Mathematics of computing** → Distribution functions; *Equational models*; • **Applied computing** → *Economics*; • **Theory of computation** → *Algorithmic mechanism design.*

Additional Key Words and Phrases: stable matching, college admission, fairness, statistical discrimination, differential correlation, continuum economy

ACM Reference Format:
Rémi Castera, Patrick Loiseau, and Bary S.R. Pradelski. 2022. Statistical Discrimination in Stable Matchings. In *Proceedings of the 23rd ACM Conference on Economics and Computation (EC '22), July 11–15, 2022, Boulder, CO, USA.* ACM, New York, NY, USA, 2 pages. https://doi.org/10.1145/3490486.3538364

REFERENCES

[1] Kenneth J. Arrow. 1973. The theory of discrimination. In *Discrimination in Labor Markets*, Orley Ashenfelter and Albert Rees (Eds.). Princeton University Press, 3–33.

[2] Itai Ashlagi, Afshin Nikzad, and Assaf Romm. 2019. Assigning more students to their top choices: A comparison of tie-breaking rules. *Games and Economic Behavior* 115 (2019), 167–187. https://doi.org/10.1016/j.geb.2019.02.015

[3] Eduardo M. Azevedo and Jacob D. Leshno. 2016. A Supply and Demand Framework for Two-Sided Matching Markets. *Journal of Political Economy* 124, 5 (2016), 1235–1268.

[4] Vitalii Emelianov, Nicolas Gast, Krishna P. Gummadi, and Patrick Loiseau. 2022. On fair selection in the presence of implicit and differential variance. *Artificial Intelligence* 302 (2022), 103609.

[5] D. Gale and L. S. Shapley. 1962. College Admissions and the Stability of Marriage. *The American Mathematical Monthly* 69, 1 (1962), 9–15.

[6] Nikhil Garg, Hannah Li, and Faidra Monachou. 2021. Standardized Tests and Affirmative Action: The Role of Bias and Variance. In *Proceedings of the 2021 ACM Conference on Fairness, Accountability, and Transparency (FAccT '21).*

[7] Jon Kleinberg and Manish Raghavan. 2018. Selection Problems in the Presence of Implicit Bias. In *Proceedings of the 9th Innovations in Theoretical Computer Science Conference (ITCS '18).*

[8] Edmund Phelps. 1972. The Statistical Theory of Racism and Sexism. *American Economic Review* 62, 4 (1972), 659–661.

ACKNOWLEDGMENTS

This work has been partially supported by MIAI @ Grenoble Alpes (ANR-19-P3IA-0003), by the French National Research Agency (ANR) through grants ANR-20-CE23-0007 and ANR-19-CE48-0018-01.