Quantum information

P1: Pure quit state: normed vector in Hilbert space

One qubit state: $|\psi_1 >= \alpha|0> + \beta|1>,\ |0> = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ |1> = \begin{pmatrix} 0 \\ 1 \end{pmatrix},\ |\alpha|^2 + |\beta|^2 = 1$

In general:

$$|\psi> = \sum_{i \in \{0,1\}^n} \alpha_i |i> \qquad || |\psi> || = \sqrt{<\psi|\psi>} = \sqrt{\sum_{i \in \{0,1\}^n} \alpha_i^* \alpha_i} = 1$$

P2: Composed systems: tensor product

$$|\psi> = |\psi_1> \otimes |\psi_2>$$

$|0_1 1_2> = |0>_1 \otimes |1>_2$

Entangled: not a product, ex: $\frac{1}{\sqrt{2}}(\alpha|00> + \beta|11>)$

P3: Measurement : Probabilistic and irreversible

Measuring with $\{P_0, P_1\}$ such that $P_0 + P_1 = I$ and $P_i P_j = \delta_{i,j} P_i$

with probability $||P_i|\psi>||^2$ classical outcome i state after measure $\frac{P_i|\psi>}{||P_i|\psi>||}$

Example:

$|\psi> = \alpha|0>_1 |\phi> + \beta|1>_1 |\phi'>$ Measured with $P_0 = |0><0|_1$ projector over $|0>$, $P_1 = |1><1|_1$ projector over $|1>$

with probability $p = |\alpha|^2$ $c = 0$ state after measure $|0>|\phi>$

with probability $p = |\beta|^2$ $c = 1$ state after measure $|1>|\phi'>$

P4: Unitary Evolution

$$U : |\psi> \mapsto U|\psi> \text{ with } U^\dagger U = I$$

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Mixed states

probabilistic distribution of states $\{(p_i, |\psi\rangle_i\} \longrightarrow \rho = \sum_i p_i |\psi_i >< \psi_i|$

General measurement

$\{M_i\}, \sum M_i M_i^\dagger = I \qquad p_i = Tr(\rho M_i)$ state after measure $\frac{1}{p_i} M_i \rho M_i^\dagger$

Trace out

$$|a >< a'|_1 \otimes |b >< b'|_2 \otimes |c >< c'|_3 = |a >_1 |b >_2 |c >_3 < a'|_1 < b'|_2 < c'|_3$$

$$Tr_2 |a >_1 |b >_2 |c >_3 < a'|_1 < b'|_2 < c'|_3 = < b'|b > |a >_1 |c >_3 < a'|_1 < c'|_3$$

A Purification of $\rho = \sum_i p_i |\psi_i >< \psi_i| \qquad |\psi >= \sum_i \sqrt{p_i} |i > |\psi_i >$

Fidelity

$$F(\rho, |\psi >< \psi|) = \sqrt{< \psi|\rho|\psi >}$$

Entropy

$$S(\rho) = -Tr(\rho \log_2(\rho))$$