

Understanding Understanding

Traces, Interruptions, and Partial Formalizations

M.M. — machine reading, checked by M.M.

M.M. — *Pragmatiquement-presque-anthropomorphismes-anthropomorphes*
rentrée 2026

August 2026

Abstract

This document is an evolving attempt to understand understanding without assuming in advance a single language in which understanding must ultimately be expressed.

The starting point is deliberately modest: traces can be transmitted, observed and interrupted; an observer may attempt to reconstruct what has been interrupted, using expectations and structures accumulated through previous observations. A stabilization is then a successful way of making sense of a partial trace, not necessarily a property already present in the trace itself.

A recurring distinction throughout the manuscript is between two bidirectional questions that should not be collapsed prematurely: what can be distinguished by an observer, and what can be instantiated as a reproducible process. The constructions developed here attempt to keep these two directions jointly visible, rather than reducing one to the other.

The purpose is not to introduce a new closed formal theory. Rather, the project develops a deliberately human-readable language of constructions, gestures, traces, interruptions, repetitions, orientations and partial reconstructions.

Different parts of this language may be anchored in different mathematical or computational frameworks, not as competing formalizations of a single underlying structure, but as local experiments on what becomes distinguishable, reproducible, or computable under different formal constraints. The constructions themselves need not already determine a common structure to which all such anchors would ultimately refer.

The project therefore proceeds from constructions toward formalizations, rather than from a fixed formal system toward its interpretations. In this sense, interruption is used as a way of avoiding premature commitment: instead of completing an observation by choosing a missing structure, one can keep the interruption and study the plurality of compatible reconstructions it permits.

1 Introduction

This document originates from an exploratory research process concerning understanding, representation, communication and formalization.

The guiding question is not simply how to construct a sufficiently powerful formal language in which everything can be represented. It is rather:

How can structures of meaning emerge from partial traces and local attempts at reconstruction, before a common formal language has been fixed?

This question matters because formalization is itself a form of stabilization. When we choose primitives, axioms, types, coordinates or inference rules, we gain a precise language, but we also decide in advance which distinctions and transformations are available to us.

The present work therefore explores a complementary direction. Instead of starting from a completed formal setting, it starts from constructions that can be perceived, manipulated, interrupted and interpreted.

The resulting language is intentionally redundant and sometimes ambiguous. This is not merely a stylistic choice. The redundancy is meant to preserve different possible readings long enough for their relationships to become visible.

2 A trace before its formalization

A trace is understood here as something transmissible which can be observed again, transformed, interrupted, or reconstructed.

It need not initially possess a prescribed geometry, algebraic structure or semantic interpretation. One possible trajectory through these constructions that can be sketched as:

$$\text{trace} \longrightarrow \text{interruption} \longrightarrow \text{attempted reconstruction} \longrightarrow \text{stabilization}.$$

But the arrow should not be read as a fixed algorithm.

An interruption does not necessarily determine a unique continuation. Different observers, or the same observer under different accumulated expectations, may construct different continuations which remain compatible with the trace available to them.

This introduces an important asymmetry between observation and reconstruction:

What is reconstructed need not be what was there before the interruption.

The point is not to replace completion by incompleteness, but to make the process of completion itself available for observation.

The distinction is essential. A formal completion may be perfectly valid inside a chosen framework while still being only one possible completion of the original partially observed construction.

3 Interruption rather than premature completion

One of the central ideas of the project is that an interruption can be more informative than an immediate completion.

Suppose that a trace suggests several possible continuations. A conventional formalization may choose one by introducing additional structure, axioms or conventions. Here we instead ask what can be retained while the continuation remains open.

The interruption therefore acts as a way of delaying a choice.

It does not mean that choices are impossible. On the contrary, it allows different choices to be compared after they have been made.

Schematically:

partial trace	$\xrightarrow{\text{interrupt}}$	compatible reconstructions
---------------	----------------------------------	----------------------------

rather than

$$\text{partial trace} \longrightarrow \text{chosen completion}.$$

This is one reason why the project does not regard incompleteness simply as a defect to be repaired.

4 Understanding as local stabilization

Understanding is used here in a deliberately operational sense.

An interpretation counts as an understanding when it becomes stable enough to be reproduced, communicated or used to continue a construction.

This does not imply that it is globally correct.

A locally stable interpretation may later fail under another interruption, another observation, or another mode of reading.

Thus:

$$\text{understanding} \neq \text{absolute identification.}$$

Rather,

$$\text{understanding} \sim \text{a locally reproducible way of distinguishing and continuing a construction.}$$

Understanding is used here as a relation that can become stable enough, locally and for a given mode of observation, to support further distinctions, communication or action.

This distinction allows the project to speak simultaneously about mathematical understanding, computational interpretation and more ordinary human forms of sense-making.

5 Two directions of distinguishability and instantiability

The constructions repeatedly distinguish two bidirectional questions that should not be collapsed into one prematurely or without justification: what can be distinguished by an observer, and what can be instantiated as a reproducible process.

These questions are not simply two descriptions of the same axis. They may constrain one another while remaining locally distinguishable. A construction may therefore be explored in both directions without assuming in advance that the corresponding transformations commute, coincide, or admit a common global completion.

The aim is not to preserve a binary opposition, but to preserve the possibility of a second cut.

The constructions repeatedly distinguish two bidirectional questions that should not be collapsed into one prematurely or without justification: what can be distinguished by an observer, and what can be instantiated as a reproducible process.

These questions may constrain and inform one another without being assumed to coincide. A distinction may be perceptible without yet being available as a stable operation; conversely, a process may be formally reproducible while the distinctions through which it was originally understood remain richer than its formal representation.

The project explores the space between these two directions. Interruption is one way of keeping this space visible: it prevents a possible continuation from being silently identified with the continuation that was actually observed, while allowing different completions to be compared for their reproducibility and compatibility.

6 Gestures, orientations, and local readings

Before being encoded as algebraic operations, some of the constructions in this notebook are deliberately described through gestures.

A direction can be read as a continuation. A boundary can be read as a separation. A repeated mark can be read as a persistence. A change of orientation can be read as a transformation. A missing continuation can be left as an interruption.

The point is not to replace mathematics by pictures.

Rather, pictures and gestures can preserve distinctions which a premature symbolic contraction would erase.

For example, a trace may admit several visually compatible regularizations: one reading may expose arrows, another boundaries or “walls”, another a sequential order, and another a spatial aggregation.

These need not be derived from one another.

They may instead be different attempts to reconstruct a common, partially undefined source.

Several formalizable structures may be reconstructions of one construction without one of them being its unique prior meaning.

7 The role of q, d, p, b

The symbols q, d, p, b are among the provisional marks used to retain recurring distinctions in the constructions. Their meaning is deliberately not fixed independently of the gestures, orientations and processes in which they occur.

They may later acquire more precise interpretations in a given formal anchoring, but their present role is to keep several possible readings available rather than to constitute a fixed primitive vocabulary.

They should not be understood, at this stage, as the axioms of a new mathematical system.

Their role is closer to that of a small vocabulary for describing recurring gestures:

- q can mark a direction of continuation or activation;
- d can mark a distinction, description, or local stabilization;
- p can mark a partition, passage, or persistence of a partially completed trace;
- b can mark a boundary, bearing, or unresolved configuration.

The precise interpretation is deliberately allowed to vary with the construction in which the symbols occur.

This apparent instability is part of the experiment. A later formalization may show that some of these uses can be unified, while others may need to remain distinct.

8 Formalization as an anchoring

The project does not oppose informal construction and formal mathematics.

It instead considers formalization as an *anchoring* of a construction within a particular framework.

A given construction may be anchored in graph theory, automata theory, algebra, category theory, logic, quantum information, or another formal setting.

Such an anchoring can provide powerful tools:

- definitions become precise;
- transformations can be composed;
- compatibility can be tested;

- invariants can be identified;
- conjectures can become theorems;
- computations can explore finite instances.

An anchoring both makes some distinctions precise and leaves others unresolved, invisible, or inexpressible within that framework.

This is not necessarily a failure: the resulting formal fragment can reveal what the chosen framework makes available, while the remainder may motivate another anchoring or a return to the construction itself.

A formalization is interesting both for what it stabilizes and for what it makes impossible to express.

Consequently, different formal frameworks should not necessarily be regarded as competing candidates for the one true formalization.

They can instead be used as different experiments on the same evolving construction.

9 Partial formalizations and shared structures

A construction can become mathematically productive before it has become mathematically closed.

A graph, an automaton, an algebraic operation or a categorical construction may capture a particular region of the process. The interest lies not only in proving results inside that region, but also in understanding which distinctions became available through the anchoring and which remain dependent on other readings.

10 Dynamic and computational perspectives

The approach naturally leads toward dynamic models.

Instead of representing only a final state, one can represent the sequence of partial observations, interruptions, reconstructions and stabilizations that lead to it.

This makes automata and related computational structures natural candidates for formalization.

The Master's work on Dynamic Automata provides one concrete example of such an anchoring: a portion of the broader exploratory language can be isolated and turned into a computational object without requiring the whole construction to have already been formalized.

This is precisely the kind of partial formalization the project seeks.

11 Quantum information and contextuality

The same perspective is relevant to quantum information.

Quantum contextuality already makes explicit that descriptions cannot always be consistently assembled into one context-independent global assignment.

The present project asks whether some of the intuitions behind such phenomena can be approached more dynamically:

what happens when the relevant description itself is partial, evolving and subject to interruption?

This does not yet constitute a new theory of contextuality.

Rather, quantum information provides one of the mathematical and physical settings in which the broader language of traces, local interpretations and compatibility can be tested.

12 What remains deliberately open

Several questions remain unresolved.

- Which constructions admit useful formalizations?
- Which apparent symmetries are genuine structures and which are merely consequences of a chosen reading?
- When can several local reconstructions be shown compatible?
- Can interruption provide a principled way of delaying a choice without merely replacing it by another hidden convention?
- Which mathematical frameworks best preserve the distinctions present in the original constructions?
- What new mathematical phenomena become visible when formalization is deliberately partial?

These questions are not gaps to be hidden by premature definitions. They are part of the research program.

13 Conclusion

This document should therefore be read neither as a finished theory nor as a collection of unrelated philosophical metaphors.

It is an attempt to construct a language in which mathematical formalization can happen progressively, locally, and in several possible directions.

One possible trajectory through these constructions can be sketched as

$\text{trace} \longrightarrow \text{interruption} \longrightarrow \text{reconstruction} \longrightarrow \text{stabilization} \longrightarrow \text{formal anchoring}.$
--

This is a possible reading of the constructions, not a prescribed order of formation. In particular, the two directions of distinguishability and instantiability need not follow this sequence, nor be reducible to it.

The constructions explored here involve at least two interacting questions: what can be distinguished in a partial trace, and what can be instantiated as a reproducible process. Interruption, reconstruction and formalization are ways of exploring their possible relations rather than successive stages of a single procedure.

The final arrow is intentionally not presented as inevitable or unique.

Different anchors may stabilize different aspects of the same construction. Some may produce rigorous mathematical results; others may expose limitations or contradictions; others may suggest new constructions.

The hope is that this process can provide a more human way of approaching formal reasoning: not by rejecting rigor, but by allowing the forms of experience, perception, gesture, partial knowledge

and reconstruction that precede formal rigor to remain visible long enough to become mathematically productive.

This text is an evolving entry point into a larger research notebook.

Note on the writing process

This document is part of an evolving research notebook. Some formulations have been developed and refined through interactive dialogue tools used as exploratory aids.

Such tools are treated as instruments for construction and reformulation, not as sources of mathematical authority.

The responsibility for the content and its interpretation remains entirely the author's.