Understanding Understanding

A Stabilized Intermediate Note toward Mathematical Formalization

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Abstract

This document is a stabilized intermediate slice of an evolving system of thought. It proposes to view understanding as the stabilization of interruptions in a transmissible trace. The goal is not to present a complete theory, but to expose a minimal structure from which mathematical, computational and interpretative perspectives may emerge.

1 Introduction

This note originates from an exploratory process aiming to understand how structure, meaning, and formalization arise.

Rather than starting from fixed objects or axioms, we adopt the following perspective:

> Understanding emerges when a dynamic process is interrupted and locally stabilized.

This document is not a finished theory. It is a *stabilized intermediate state* of an evolving research notebook.

2 Core Intuition

We propose the following minimal scheme: we start from pairs of marks, which may act either as candidates for interruption or as traces left by one.

Local interpretative directions attempt to align and stabilize these pairs, but such attempts are not guaranteed to succeed.

This introduces an intrinsic space of alternatives, where both successful and failed stabilizations contribute to the emergence of structure.

An interruption is not an event, but a relation whose stabilization is not guaranteed.

Rather than describing only what is stabilized, we also account for the complement: what is not realized, but remains accessible through the dynamics of interruption.

Locally we can view:

$$\texttt{trace} \to \texttt{interruption} \to \texttt{stabilization} \to \texttt{structure}$$

- A **trace** is a transmissible sequence or process.
- An **interruption** is a local rupture that forces a distinction.
- A **stabilization** is a resolution or interpretation of this rupture.
- A **structure** is what persists across such stabilizations.

In this view, meaning is not given in advance but constructed through interruptions.

Failure is not outside the system; it is one of its fundamental modes.

A trace does not come with a geometry; geometry is how we choose to read it.

3 Primitives

We isolate a minimal set of primitives:

- **Interruption**: a local break in a process.
- **Trace**: a medium in which transformations propagate.
- **Repetition**: the possibility of re-instantiating a pattern.
- **Transmission**: the propagation of a trace across contexts.

These primitives are not fully formalized here. They are intended as a minimal basis for further development.

4 Duality of Operations

We introduce two primitive gestures:

- Composition, denoted by |
- Separation, denoted by -

These operations are not yet defined formally, but can be seen as the basic mechanisms by which traces are combined and distinguished.

They correspond to two complementary actions:

- building continuity,
- introducing distinction.

5 Dynamic Perspective

We do not consider static objects, but evolving processes.

An interruption transforms a trace and creates a space of possible interpretations.

> Structure is not primary; it is the result of stabilized transformations.

This suggests a view of mathematics where:

- invariants are stabilized traces,
- computation is guided interruption,
- meaning emerges from constrained dynamics.

6 On Interpretation

Interpretation is unavoidable.

An interruption does not resolve itself; it requires a choice of stabilization.

Thus:

> To apply a system is already to interpret it.

This implies that understanding cannot be reduced to mechanical execution.

7 Ethical Perspective

The necessity of interpretation has ethical consequences.

> A system that suppresses interruption suppresses responsibility.

Any formal or computational system that removes the possibility of questioning also removes the possibility of responsible use.

In this framework:

- understanding implies responsibility,
- application without interpretation is a structural failure,
- acknowledging uncertainty is a constructive act.

8 Toward Formalization

This note does not provide a complete formal system.

However, it suggests that:

- formal structures may be derived from interruption dynamics,
- stability conditions may define mathematical objects,
- different domains (logic, computation, physics) may be unified through a theory of transformations and interruptions.

9 Sequential and Constructive Perspective

We adopt a sequential viewpoint: structures are not given a priori as families, spaces, or distributions, but emerge from sequences of interactions.

> Agents, space, and geometry are not primitives, but compatible projections of a common underlying structure of traces and interruptions.

9.1 From Sequences to Structure

We consider that the primary object is a *trace*, understood as a sequence of transformations or interactions. Structure arises when such a trace is interrupted and locally stabilized.

> Understanding corresponds to a stabilized interpretation of an interruption.

In this perspective, global structures are not assumed but progressively constructed.

9.2 Interaction without Primitive Agents

We do not assume the prior existence of agents exchanging messages.

Instead, we consider a network of traces undergoing interruptions and recompositions.

What may later be interpreted as "agents" corresponds to stable regions within this network, where interactions appear as exchanges relative to a chosen partition.

> A system of agents is a plausible partition of a structure of traces.

9.3 On Probability

Probability is not taken as primitive. It appears as a secondary notion, derived from the repetition of stabilized patterns.

> A probability distribution summarizes repeated instances of a stabilized model, but does not capture the local dynamics of interruption.

As such, probabilistic descriptions may obscure the fine structure of interpretation and transformation.

9.4 Emergence of Space Through Interaction

We propose that space and geometry are not predefined, but co-constructed through interaction.

In particular, we consider processes of message passing, where interactions can be interpreted, under a given partition, as exchanges between emergent agents that progressively build representations of one another.

> The space in which representations live is shaped by the interactions that produce them.

This suggests that geometry, distance, and even metric structure may emerge from the dynamics of communication.

9.5 Subjectivity and Local Uncertainty

In such a framework, an agent does not have access to a global description of the system.

Instead, it constructs partial representations based on local interactions. This induces a form of intrinsic uncertainty:

- interpretations depend on local stabilizations,
- different agents may construct different but compatible structures,
- global consistency is not directly observable.

This perspective suggests a link between subjectivity and entropy, not as a global quantity, but as a measure of local indeterminacy within the process of interpretation.

9.6 Indistinguishability of Understanding

An important consequence is that certain distinctions may become experimentally inaccessible.

> An agent embedded in the system may not be able to determine, from local observations alone, its position in the global structure.

In particular, the distinction between *having the impression of understanding* and *actually understanding* may correspond to configurations that are locally indistinguishable.

This reflects a deeper principle:

> Understanding is not an absolute property, but a stabilized relation within a dynamically constructed space.

10 A Minimal Transition System

We introduce a minimal formal system intended to capture the roles q, d, p, b as elementary transformations of a trace.

10.1 Traces and Interruptions

A *trace* is a finite sequence

$$\tau = (x_1, x_2, \ldots, x_n)$$

whose elements represent events or marks.

An *interruption* is a position i in the trace where a transformation may occur.

10.2 States and Roles

We consider a system of states (τ, r) where:

- τ is a trace,
- $r \in \{q, d, p, b\}$ is a role.

Each role corresponds to a mode of interpretation of the trace.

10.3 Transition Rules

We define elementary transitions:

Temporal role (q).

$$(\tau, q) \to (\tau', q)$$

where τ' extends τ by appending a new element:

$$\tau' = (\tau, x)$$

This models ordering and indexing.

Descriptive role (d).

$$(\tau, d) \to (\tau', d)$$

where τ' is obtained by locally rewriting a segment of τ:

$$(x_i, x_{i+1}) \mapsto y$$

This models local stabilization.

Partition role (p).

$$(\tau, p) \to ((\tau_1, \tau_2), p)$$

where τ is split into subtraces. This models separation and distinction.

Global role (b).

$$(\tau_1, \tau_2, \ldots) \to (\tau', b)$$

where multiple traces are merged into a larger structure. This models global integration.

10.4 Interpretation

These transitions do not define a fixed computation, but a space of possible transformations.

A reasoning process corresponds to a path in this transition system.

Different roles emphasize different aspects:

- q organizes,
- d stabilizes,
- p separates,
- b integrates.

10.5 Remark

This system is intentionally minimal. It serves as a first approximation of a more general framework where traces, interruptions, and interpretations interact.

10.6 Binary Interpretation of Roles

We propose a minimal binary interpretation of the roles q, d, p, b as elementary transformations on a two-state system $\{0, 1\}$:

Role	Transformation	Interpretation
q	$0 \mapsto 1$	activation / reception
d	$1 \mapsto 1$	stabilization / persistence
p	$1 \mapsto 0$	release / failure
b	$0 \mapsto 0$	latency / openness

These four roles correspond to distinct ways of responding to an interruption.

- d represents successful stabilization,
- p represents failure or incomplete transmission,
- q explores the complement by activating new possibilities,
- b maintains unresolved configurations.

Reasoning can be viewed as a composition of such elementary transformations, where both successful and failed stabilizations contribute to the process.

10.7 Duality of Reading: Flow and Aggregation

The same trace may admit multiple modes of interpretation.

On the one hand, it can be read sequentially, as a flow:

- a progression in time,
- a succession of transformations,

- a narrative structure.

On the other hand, it can be read as an aggregated structure:

- grouped into blocks,
- reorganized into patterns,
- interpreted as a spatial configuration.

> A trace is not inherently linear or spatial; these are modes of reading induced by stabilization.

Different decompositions (e.g. grouping elements, interleaving, or partitioning into substructures) give rise to distinct but compatible interpretations.

This suggests a duality between:

- flow (temporal, sequential),
- aggregation (spatial, structural).

10.8 Flow and Aggregation

The four roles q, d, p, b can also be interpreted as mediating between two complementary modes of reading a trace.

On the one hand, a *flow-oriented* reading emphasizes sequential progression:

- q activates and advances the trace,
- d stabilizes local configurations along the sequence.

On the other hand, an *aggregation-oriented* reading emphasizes structural organization:

- p separates and partitions the trace into components,
- b maintains unresolved or global configurations.

> The same trace may be read either as a flow or as an aggregated structure, depending on how roles are composed.

This suggests that temporal and spatial interpretations are not primitive, but arise from different modes of organizing transformations.

> Reasoning emerges from the composition of multiple modes of reading, such as flow and aggregation, rather than from their opposition.

Operational intuition. The roles q, d, p, b may be understood as elementary modes of interaction with a trace:

- q: extend or activate,
- d: stabilize or transform,
- p: read or separate,
- b: wait or maintain openness.

These roles do not define a fixed computation, but a space of possible interactions with a structure.

11 Conclusion

This document is a partial stabilization of an ongoing exploration.

It isolates a minimal set of ideas:

- interruption as a primitive,
- trace as a medium,
- stabilization as understanding,
- interpretation as necessary.

These elements suggest a broader program, where mathematical structures are defined not only by invariants, but by their stability across multiple interpretative modes.

qdpb-Open Mathematics. We propose to view this framework as a form of *qdpb-open mathematics*, where openness is not generic but structured by the four roles q, d, p, b.

Rather than assuming globally closed and fully specified structures, this perspective focuses on locally stabilized configurations, organized through these four modes of transformation.

In this perspective, classical formal theories appear as particular, localized regimes of stability within a broader space of transformations.

> Universality is not assumed globally, but recovered locally through stabilization.

The openness considered in this work is not arbitrary. It is structured by four fundamental roles q, d, p, b, which organize the space of possible transformations and interpretations.

> Openness is not the absence of structure, but the presence of a minimal organizing scheme.

These roles define a constrained form of openness, where local consistency is maintained, while global closure is not assumed.

This text should be read as an entry point rather than a complete theory. Further work will refine these notions and explore their formal consequences.

Note on the writing process

This document is part of an evolving research notebook. Some formulations were refined through interactive dialogue tools, used as exploratory aids to stabilize intermediate expressions.

The responsibility for the content and its interpretation remains entirely the author's.