Understanding Understanding

Bi-expressible, bi-audible, interruptible: possible traces, anthropomorphism, and partial formalizations

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Abstract

This project explores how something may come to be approached as a possible trace, interrupted, transmitted and locally reconstructed, without assuming in advance a single language in which every relevant distinction must eventually be expressed.

The earlier entry point considered traces, interruptions, possible reconstructions and local stabilizations. This version asks what is already being assumed when something is identified as a trace, and what may remain possible when the reconstruction of such an instance is itself interrupted.

Calling something a trace may seem to assert an existence: something traced, a tracer, a material or medium capable of being traced, and some localization at which these become related. Yet even this is not enough to explain how what is locally produced or noticed is transmitted to an interpreter, by a messenger or through a medium, and then validated, tagged, questioned or asked about again: what, when, and why?

The exploratory expression *a split that leaves two further splits available* describes a recurring intuition without attributing life or intrinsic agency to the resulting branches. Anthropomorphism makes living a question to be examined, not a fact established by the vocabulary. The manuscript develops this intuition through gestures, decorated marks, orientations, four provisional roles q, d, p, b, and experiments with algebraic composition.

The point of interrupting interruption is not to postpone rigor indefinitely. It is to delay the axiomatization of beliefs as if they were already statable, separable and simplifiable, without first asking for whom, by whom, through which transmission, and under what conditions. Formalization may make distinctions reproducible; it may also hide the choices and singularities through which they became available.

This is not a closed theory. It is an attempt to keep several mathematical, physical, philosophical and linguistic constructions available for comparison without assuming that they already belong to one complete description.

1 Before calling something a trace

Understanding rarely begins with a complete description. We observe, remember, transmit and reconstruct from what may be treated as traces, although the status of these traces is itself part of the question.[1]

$$\text{possible trace} \longrightarrow \text{interruption} \longrightarrow \text{reconstruction} \longrightarrow \text{local stabilization}.$$

[1] This entry point is intended to be readable on its own, as an invitation to interact with the questions rather than accept a summary of them. For an earlier version of the evolving project, see https://lig-membres.imag.fr/mhalla/understanding/understandingmachineoctober.pdf.

This sequence is useful, but it should not be mistaken for an algorithm or a prescribed chronology. A reconstruction can itself become the object of a new interruption. A later reading may reveal that what had seemed to be a single point, a boundary or a common object depended on a choice that was not previously visible.

The present exploration therefore gives particular attention to the continuation of a construction after it has been divided. The important question is not only what lies on either side of a proposed division, but what allows either side to be identified, transmitted and revisited as a side at all.

Even an instance of a possible trace seems to invite a chain of commitments: a tracer, something traced, a traceable material or medium, a localization that joins these in some way, and a messenger or transmission through which an interpreter comes to encounter it. The interpreter may then validate it, tag it, reject the tag, or ask again: what is this, when did it become available, and why should it count? None of these links need be settled in advance; nor should their mere naming be mistaken for their justification.

A proposed split may leave two possibilities of further splitting. Their contents need not be identical across interpretations. What may persist is not a living branch, but the availability of another question or continuation. This is a working intuition, not yet a mathematical invariant.

The point is not to forbid the language of life, but to keep its attribution interruptible. Anthropomorphism makes living a question: who or what is being treated as alive, by whom, through which signs, and with what consequences?

2 What is taken to be a trace is not exhausted by its reading

The notebook continues to use points, strokes, arrows, brackets, boundaries, rings and fingers as ways of making interruption perceptible. Calling any of these a trace is already an interpretive move, not a neutral report of an independently established object.

These marks are not simply illustrations of operations already defined elsewhere. Their orientations and arrangements may participate in the construction of the distinctions one subsequently attempts to formalize; but the passage from mark to trace, and from trace to evidence, remains open to question.

A point may be understood as a location, but its location may depend on what the pointing gesture hides. A boundary may separate two regions while also making their relation visible. A repeated mark may be interpreted as persistence, or as a sequence of transformations whose intermediate differences disappear from the final representation.

The same trace can therefore admit different regularizations. One reading may privilege direction, another a boundary, another a sequence, and another a spatial arrangement. These readings may be compatible without being identical, and their compatibility need not imply the existence of one complete description from which all of them follow.

The project consequently distinguishes between what is offered or encountered and what an interpreter stabilizes as a trace. A successful reconstruction is not necessarily a recovery of something that existed in the same form before the interruption. It is a way of making an encounter sufficiently intelligible to support further distinctions, communication or action—while leaving open how the encounter was localized, transmitted and accepted as relevant.

This distinction becomes particularly important when a formal representation is introduced.

A formalization can reveal a structure that was previously difficult to express, but it can also conceal the gestures, choices and alternative readings that made the structure available in the first place.

3 Four provisional colorations

The symbols q, d, p, b remain a small experimental vocabulary. They are associated, provisionally, with four anthropomorphized ways of reconstructing a trace:

- q: Qudsia, exploring physical supports, manifestations and possible continuations;
- d: Dolores, exploring mathematical distinctions, structures and reproducible relations;
- p: Perséphone, exploring philosophical links, attribution and questions of identity or difference;
- b: Bénédicte, exploring natural language, narration and the motivations that make a reading continue or change direction.

These roles should not be treated as fixed types or as four independent domains. They are ways of making visible the different intentions that may be attributed to a reader or interpreter.

Their provisional anthropomorphism is deliberate. Giving a role a name can make an attributed intention easier to recognize, question and revise. It does not establish that the corresponding object possesses an intrinsic intention. Nor does the use of four names prove that all interpretations can be reduced to four fundamental categories.

The roles may change their significance with the construction in which they occur. Even their apparent order may depend on whether one follows the actor's imagined temporal progression, the reader's reconstruction, or the direction imposed by the written trace.

The question is therefore not simply how to assign meanings to four symbols. It is how to retain the possibility that the same symbols may participate in different readings without making their differences disappear.

4 Pairings, filtering and the possibility of further division

The notebook explores sixteen pairings associated with the four roles, while investigating twelve pairings as possible carriers of meaning.

The status of the twelve is not yet fixed by a general rule. It would be premature to decide whether the remaining four are impossible, redundant, unproductive, or simply not yet understood. Their absence may itself be informative, but its interpretation remains open.

Earlier experiments with protoforms provide a related observation. Four functions may be filtered differently depending on the interpretation. The filtering is not always the same, yet a split leaving two further splits alive appears to recur.

This suggests a possible distinction between the content of a filtering and the form of continuation it preserves. An invariant, if one exists, may concern neither the identities of the functions nor the precise pairings retained, but the possibility of further division.

That remains a conjectural direction rather than an established result. The next mathematical step would be to determine which properties of the protoforms are actually preserved under

changes of interpretation, and which similarities are consequences of the particular examples examined so far.

The purpose of this question is not to force all the pairings into a single table. A table is useful as a trace of the exploration; it should not automatically become the authority that determines which continuations are legitimate.

5 A composition that seems to do nothing

The recent algebraic experiments investigate expressions that return to an identity while exposing several intermediate factors.

$$U_{-}^{2}\, U_{|}^{2}\, U_{-}^{2}\, U_{|}^{2} = I.$$

The notation is provisional: the equality requires the relevant algebraic assumptions about the operators and their composition. The exploratory point is that the expression displays four factors even though its overall result is the identity.

The global result may therefore be neutral while the path of composition retains distinctions. Interrupting the expression at different locations gives different prefixes, suffixes or intermediate reconstructions. These may be associated experimentally with q, d, p, b, without assuming that the four roles are already determined by the algebra.

This is one possible meaning of a *bi-operant that seems to do nothing while opening into four*: a composition whose final reading does not reveal everything that can be investigated in its construction.

Three questions must remain distinct:

1. Under which algebraic conditions does the composition actually equal the identity?
2. Which intermediate distinctions remain available when the expression is interrupted?
3. What justifies interpreting those distinctions through the four roles?

The first is an algebraic question. The second concerns the structure of the construction and its possible readings. The third concerns the relation between the notation and the meanings attributed to it.

Their eventual connection would be interesting, but none of the three should be silently substituted for the others.

6 Quantum information as a possible anchor

Quantum information offers concrete examples in which a small collection of operations can generate a rich family of transformations, while measurement-based computation illustrates how a resource state and a restricted set of measurements can support universal computation.

These examples motivate questions about composition, intermediate structure and the difference between an operation's final effect and the construction through which that effect is obtained.

They do not, by themselves, establish the proposed correspondence between the four roles, the sixteen pairings and the algebraic expressions in the notebook. The purpose of the comparison is exploratory: to identify which parts of the construction could be anchored in a familiar mathematical setting and which distinctions such an anchoring would preserve or erase.

A formal anchor may make a local construction precise enough to test. It may also reveal that two apparently similar readings are mathematically different, or that a desired identity does not hold under the proposed assumptions. Such a result would not necessarily invalidate the original exploration. It would locate the limits of a particular reconstruction.

The ambition is not to obtain universality merely by naming four roles or choosing a cycle of four factors. It is to investigate whether the relations between the factors, their possible compositions and the interpretations of their traces can support useful structures.

7 Formalization, contraction and the loss of singularities

The notebook returns to a tension between regularization and the preservation of difference.

Regularization can simplify communication by reducing a construction to a finite number of recognizable distinctions. This contraction may make comparison, transmission and reproducibility possible. Yet it can also hide singularities by making different cases appear interchangeable.

The working notion of *addictibility* points to this tension. An interruptible access to traces may be beneficial, but it should not be confused with an unlimited repetition that no longer permits meaningful reassessment. The proposed safeguard is not a guarantee of truth: it is the possibility of planned checks, reconsideration and interruption.

The distinction is important because a representation can become easy to transmit while becoming difficult to question. A local stabilization may acquire the appearance of an independent object, and its subsequent use may carry consequences that are no longer attributed to the original choices.

The notebook expresses this concern in the claim that manufacturing linearity reduces the inexpressible to an almost-nothing that can be centralized and made responsible for an adjunction.

One possible reading is that linearization isolates a small fragment that can be transmitted as though it were sufficient to support a larger relation. The fragment becomes useful precisely because it is separable and repeatable. But its usefulness does not prove that it contains everything that was present before the separation.

Formalization should therefore be examined both for what it makes reproducible and for what it renders invisible.

8 Division and the justification of inclusion

A related problem appears in the attempt to justify equitable division.

A simplified model distributes a quantity X among H recipients, each receiving Y, while retaining a remainder K:

$$X - K = HY.$$

Introducing another recipient changes the arithmetic. But the arithmetic alone does not determine whether the new recipient should be included, what constitutes a fair share, or how the remainder should be interpreted.

The example separates at least three questions: whether the calculation is correct, whether the rule of division is justified, and whether the conditions for inclusion are acceptable.

This distinction connects the mathematical experiments to the broader question of responsibility. A rule can be internally consistent while concealing the decision that determines who counts as a recipient. Conversely, making the decision explicit does not automatically settle every question about fairness.

The point is not to replace mathematical reasoning with a moral conclusion. It is to keep visible the transition from a formal relation to the use made of that relation.

9 Anthropomorphism, responsibility and the limits of prediction

The anthropomorphized roles also make it possible to ask who is responsible when a representation is used to predict, classify or justify an action.

An anthropomorphism may acquire predictive power: it can support estimates, and those estimates can become inputs to further estimates. Yet its predictive success may be limited by what it cannot understand or represent.

The notebook asks how a language, an institution, an instantiation and a reader become linked in a chain through which a claim comes to be treated as objective. Each link may contribute to the appearance that the final consequence simply follows from the construction, even though the conditions of validation and localization were themselves established somewhere.

This does not imply that every formal result is merely subjective. It asks instead how a result becomes transmissible, recognized and actionable, and where responsibility lies for the assumptions that make this possible.

The distinction between doing and wanting to do is particularly important here. A command, a recommendation or a representation can distribute responsibility among those who accept, enact or reinterpret it. That distribution should remain open to examination rather than disappearing behind the apparent neutrality of the transmitted object.

10 Understanding as a revisable relation

The project does not seek to preserve ambiguity for its own sake. It seeks ways to make constructions explicit without deciding in advance every continuation that another interpreter might discover—or assuming that the relevant beliefs have already been stated in a form that can safely be simplified.

An interpretation can be sufficiently stable to support communication or action while remaining revisable. Its justification may permit continuation, restrict it, or require reconsideration of the conditions under which the interpretation was accepted.

These are different possibilities. Justification does not always mean permission to continue, and interruption does not always mean prohibition. Sometimes it makes a previously hidden choice available for examination.

The present exploration therefore treats understanding as a revisable relation among possible traces, their production or localization, their transmission, interpreters and possible continuations rather than as the complete identification of a fixed object. A shared understanding may emerge through local stabilizations, but its emergence does not guarantee that all participants have reconstructed the same thing.

This is also why the project does not assume a single final language. Mathematics, physics, philosophy and natural language may each provide ways of transmitting responsibility between people, while exposing different aspects of the construction.

The translations between them may be productive even when they are not lossless. Their differences can reveal what a particular formalization has made visible, what it has suppressed, and what might require another reconstruction.

11 What remains open

Several questions now stand out:

- Can the persistence of a split leaving two further splits alive be characterized independently of the particular protoforms that suggested it?
- Is there a principled relation between the sixteen pairings, the twelve pairings under exploration and the different filters induced by interpretation?
- Under what conditions can a composition that returns to the identity retain meaningful distinctions in its intermediate traces?
- Which parts of these constructions can be anchored in algebra, graph theory, automata or quantum information without erasing the distinctions that motivated them?
- How can regularization simplify communication while keeping the loss of singularities visible?
- When does justification support a continuation, when does it limit one, and when does it require a return to the conditions of interpretation themselves?
- How can a representation remain sufficiently explicit to be checked by another person without becoming a device that determines in advance what that person is allowed to understand?

These questions do not form a linear sequence of tasks. Each may reopen the construction from a different direction. Some may eventually lead to formal results; others may reveal that an apparent common structure must remain a family of related but distinct constructions.

12 An open continuation

The present entry point does not close the movement from traces to formalization. It returns to the trace after attempting several formal readings.

A composition may be an identity and still offer something to explore. A division may be arithmetically correct and still leave inclusion unjustified. A local interpretation may be reproducible without being the only possible reconstruction. A representation may communicate successfully while concealing the conditions under which it became communicable.

The recurring question is how to preserve these differences without giving up the possibility of shared constructions—and how to ask, before axiomatizing a belief, who can state it, who can simplify it, for whom, and by what chain of localization and transmission it has become available.

Cultivating ambiguity does not require abandoning rigor. It may instead mean allowing different readings to remain available long enough for their compatibility, incompatibility and consequences to become visible. A useful formalization is one that clarifies what it can establish while leaving room to identify what it cannot yet express.

The project remains a walk through gardens of partially meaningful constructions. Some paths may become stable enough to support a mathematical result; others may lead back to the gestures and interruptions from which they began. Their order is an order of exploration, not a claim that one path must be the origin or destination of all the others.

Perhaps the aim is to take the time to understand when and how one can turn a duty to act into a demand that others act, while cultivating the diversity of ways of learning and responding to adversity. The possibility of understanding together may depend not only on what can be transmitted, but also on preserving the opportunity to interrupt, question and reconstruct what has been transmitted.

This text is an evolving entry point into a larger research notebook. It is a machine-assisted reading and reformulation, checked and discussed by the author. The constructions remain open to correction, reinterpretation and further formalization.