Understanding Understanding

Traces, Interruptions, and Contextual Stabilizations

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Abstract

This document is a compact, humanly checked entry point into an evolving research project on understanding, representation and formalization. It asks how structures of meaning can emerge from partial traces and local attempts at reconstruction before a common formal language has been fixed.

An interruption does not merely hide what comes next. It also leaves undecidable what, in interrupting, has been cut short. And once instantiated, an interruption is itself interruptible.

A central methodological choice is to interrupt premature completion. Rather than silently filling a missing part with one preferred structure, the project keeps several possible reconstructions in play and asks how they are imagined, located, compared and made acceptable through sequences of interruptions. These sequences can themselves be interrupted or nested within other sequences.

The project does not treat mathematics or formal language as a final ontology that resolves this process. They too are gestures, representations and interruptions: ways of making some distinctions available while potentially concealing others. Recent work arranges four recurring roles into a garden of local narrations and asks how distinct equation-based theories might arise as projections of related constructions, and under what conditions they could remain compatible without becoming identical. A more concrete attempt now connects matrix insertions, open and completed shapes, and a default spatial ring, then asks whether adjacent increasing and decreasing rules can generate a synchronized geometry rather than presupposing one.

1 A question before a theory

The project originates in an exploratory research process concerning understanding, representation, communication and formalization.

> How can structures of meaning emerge from partial traces and local attempts at reconstruction, before a common formal language has been fixed?
>
> A more specifically mathematical version is already taking shape: when can distinct equation-based theories be understood as projections of related constructions, and what would make those projections compatible without making them equivalent?

Choosing primitives, types, coordinates, axioms or inference rules changes which distinctions and transformations can be made. But this choice is not a view from outside the process: the formal language is itself a gesture that can be interpreted, interrupted and reconstructed.

The question is therefore not simply how a construction can be anchored in mathematics. It is also what the act of formalizing does to the construction, what it makes distinguishable, and what it renders indistinguishable or inexpressible. A formal framework may be useful without being the solution toward which the initial gesture was secretly moving.

2 A trace before its formalization

A trace is something that can be transmitted, observed again, transformed, interrupted, or reconstructed. It need not initially possess a prescribed geometry, algebraic structure or semantic interpretation.

A possible trajectory is

$$\text{trace} \longrightarrow \text{interruption} \longrightarrow \text{attempted reconstruction} \longrightarrow \text{stabilization}.$$

But this arrow is not a fixed procedure. An interruption does not necessarily determine a unique continuation. Different observers, or the same observer after different histories of observation, may construct different continuations that remain compatible with the trace available to them.

> What is reconstructed need not be what was there before the interruption.

A reconstruction is nevertheless constrained by the trace, accumulated expectations, available operations, and what can subsequently be reproduced or communicated.

3 Interruption rather than premature completion

Suppose a trace suggests several possible continuations. A conventional formalization may choose one by introducing additional structure, axioms or conventions. Here one can instead ask what remains available while the continuation is left open:

$$\boxed{\text{partial trace} \overset{\text{interrupt}}{\longrightarrow} \text{compatible reconstructions}}$$

rather than

$$\text{partial trace} \longrightarrow \text{chosen completion.}$$

The interruption delays a choice without forbidding it. Once several choices have been made, they can themselves become objects of comparison.

Repeated interruption can therefore be constructive: it may reveal which features persist across reconstructions and which depended on a particular completion.

4 Understanding as local stabilization

Understanding is used here in an operational sense. An interpretation counts as an understanding when it becomes stable enough to be reproduced, communicated, or used to continue a construction. This does not imply that it is globally correct:

$$\text{understanding} \neq \text{absolute identification.}$$

Rather,

$$\boxed{\text{understanding} \sim \text{a locally reproducible way of distinguishing and continuing a construction.}}$$

The word "locally" is essential. A stabilization may work for one observer, community, language, task or mode of observation, and later fail under another interruption.

The question consequently becomes not only "what is the object?" but also: what distinctions can be maintained, communicated and acted upon here?

5 The point as an achievement

A recurring temptation in formalization is to begin with points. Points are useful: they are precise, manipulable and easy to place in a common mathematical space. But their usefulness can conceal the work required for several observers to treat something as the same point.

The project therefore experiments with a weaker starting intuition:

> What we call a point may be a local stabilization among contextual views that have become sufficiently compatible.

This is not a rejection of points or geometry. It is a reminder that the ideal point is an ideal construction. A point can be extremely effective once the context has been fixed. The project asks what happened before that fixing, and what becomes visible if the stabilization is interrupted.

In this sense, a point may be understood less as the primitive from which relations emerge than as something that can emerge from relations. The same question applies to boundaries, common objects, names and other apparently elementary entities.

6 A family of paths rather than a privileged route

The project repeatedly produces several mathematical or conceptual continuations from the same construction. This proliferation is not treated simply as a defect of an incomplete theory. A construction may give rise to several directions:

$$M \longrightarrow M_1,\ M_2,\ M_3,\ M_4, \ldots$$

The task is not necessarily to decide immediately which branch is "the real" one. It is to explore relations between branches: inclusion, translation, duality, approximation, specialization, compatibility, incompatibility, or perhaps no useful relation at all.

The resulting structure is better approached as a family of paths than as a single route whose branches all descend from one privileged root. The image of a forest can suggest that the main task is to classify the trees. Here the emphasis is instead on the possibility of visiting a dynamic construction one direction at a time, and on what becomes visible when paths are compared.

The order in which branches are explored is an order of research, not an ontological ranking. One branch may be explored first because it illuminates another, without thereby becoming the origin or destination of the others.

7 Gestures, orientations and representations

Before being encoded as algebraic operations, some constructions are described through gestures. A direction can be read as a continuation. A boundary can be read as a separation. A repeated mark can be read as persistence. A change of orientation can be read as a transformation. A missing continuation can remain an interruption.

Pictures and gestures are not intended to replace mathematics. They can preserve distinctions that a premature symbolic contraction would erase.

A trace may admit several visually compatible regularizations: one reading may expose arrows, another boundaries or walls, another sequential order, another spatial aggregation. These readings need not be derived from one another. They may be different reconstructions of a partially undefined source.

> Several formalizable structures may be reconstructions of one construction without one of them being its unique prior meaning.

This is why intermediate diagrams and assemblages remain visible: they record stabilizations that may later become useful from another direction.

8 The provisional roles q, d, p, b

The symbols q, d, p, b recur in the constructions. Their meaning is deliberately not fixed independently of the gestures, orientations and processes in which they occur. They are not currently presented as the axioms of a new mathematical system, but as a vocabulary for recurring distinctions.

One possible reading is:

- q can mark a direction of continuation or activation;
- d can mark a distinction, description or local stabilization;
- p can mark a partition, passage or persistence of a partially completed trace;
- b can mark a boundary, bearing or unresolved configuration.

The precise interpretation may vary with the construction. This instability is deliberate. A later formalization may show that some uses can be unified while others must remain distinct.

The roles should therefore not automatically be read as fixed types. The same participant or construction may occupy different roles relative to different processes or contexts.

9 A garden of four narrations

The latest exploratory pages arrange the recurring roles around four names: Qudsia (q), Dolorès (d), Perséphone (p) and Bénédicte (b). This is not yet a definition of four mathematical types. It is a way to narrate four partially distinguishable ways in which a construction may begin, be localized, be transmitted and become understood.

The words *inni*, *quandou*, *etou* and *pouar* act as local markers in this developing language. In particular, *inni* names a responsibility for introducing a definition or memory without claiming complete knowledge of what it will later mean; *quandou* marks the local instantiation of a form to be recognized and retranscribed. These are project-specific uses, not claims that the words have these meanings in ordinary language.

Each role can narrate a family of paired traces that may be drawn toward a centre or outward, and refined by inserting further points. A default pointer, a default direction of continuation, and a default placement make one first reading possible; they are conveniences for beginning, not universal choices. The local narrator may know enough to continue without knowing whether the partial geometry should be read globally as a square, a rhombus, or another compatible construction.

The emerging image is therefore not a completed diagram but a garden of four narrations. When one narration is referenced, it can temporarily serve as a relative time or *tempsTempérature* for the other three, whose progress is then read as connected weights on a dial. This is an exploratory construction: its mathematical coherence and invariants remain to be established. Its present purpose is to keep localization, direction, memory and transmission visible while resisting premature identification of a single global object.

10 Two directions of distinguishability and instantiability

The constructions repeatedly distinguish two questions:

1. what can be distinguished by an observer?
2. what can be instantiated as a reproducible process?

They should not be collapsed prematurely. A distinction may be perceptible without yet being available as a stable operation. Conversely, a process may be formally reproducible while the distinctions through which it was originally understood remain richer than its formal representation.

The project therefore explores the space between distinguishability and instantiability. Interruption is one way of keeping this space visible. The aim is not to preserve a binary opposition, but the possibility of a second cut: after a first distinction has been made, what additional distinction becomes possible by examining the relation between the two directions?

11 Formal languages as interruptible representations

Mathematics is not introduced as a final language in which the earlier construction receives its solution. A formal notation can be a powerful stabilization, but it remains one of the gestures available for making a construction readable, reproducible or transmissible. It can itself be interrupted.

A letter is not simply a letter once its identity is made interruptible. What seemed to be the same sign may be reread as a trace of different gestures; what seemed different may remain indistinguishable within the representation at hand. The formal language then does not necessarily solve the distinction. It may represent the limits of an anthropomorphic attempt to distinguish two referents that have nevertheless been differentiated in two ways.

This does not establish that the referents are identical, nor that they must be different in every possible framework. It says that the current representation does not sustain the distinction in the way its user might have wanted. A formalization can thus reveal both the distinctions it enables and the limits of the act of distinguishing.

> A formal language is not outside interruption: it is one of the things that can be interrupted, and one of the ways an interruption can be represented.

12 Partial formalizations

A construction can become mathematically productive without becoming mathematically closed. A graph, automaton, algebraic operation or categorical construction may express one locally stabilized distinction. It can then serve as material for another interruption: which distinctions became available through this representation, and which depend on other readings?

A formal attempt that does not provide the expected unification may still be valuable if it shows that two apparently similar constructions behave differently, or that a framework silently introduced a distinction absent from the gesture that prompted it. Conversely, the framework may fail to distinguish what the original gesture treated differently.

The movement is therefore not a ladder toward a final formal anchor. One possible local sequence is

$$\text{gesture} \to \text{trace} \to \text{attribution} \to \text{representation} \to \text{interruption of the representation},$$

but any stage can be revisited, and the sequence itself can be embedded in another. The point is not to privilege this order, but to keep visible that formalization participates in the process it represents.

13 Dynamic and computational perspectives

The approach naturally leads toward dynamic models. Instead of representing only a final state, one can represent sequences of partial observations, interruptions, reconstructions and stabilizations.

This makes automata and related computational structures natural candidates. The Master's work on Dynamic Automata provides one concrete example of such an anchoring: a portion of the broader exploratory language can be isolated and turned into a computational object without requiring the whole construction to have already been formalized.

This is precisely the kind of partial formalization the project seeks. The point is not that automata are the final language of understanding, but that they can test what happens when some dynamic distinctions are made explicit and reproducible.

14 Quantum information and contextuality

The same perspective connects naturally with quantum information. Quantum contextuality already makes explicit that descriptions cannot always be assembled into one context-independent global assignment.

The present project asks whether some of the intuitions behind such phenomena can be approached more dynamically:

What happens when the relevant description itself is partial, evolving and subject to interruption?

This is not proposed as a new theory of contextuality. Quantum information provides a demanding mathematical and physical setting in which traces, local interpretations and compatibility can be tested.

One question being explored is what changes when the link between correlations and probabilities is itself treated as something that may be interrupted. This is a research direction, not a claim that the broader project has already supplied a new theory of contextuality.

A related open intuition is that the approximability of a description through its definability may be orthogonal to a chain of measurement–observation refinements. This remains a direction to examine, not an established result. Quantum information supplies a demanding setting in which local descriptions, compatibility and the limits of global reconstruction can be tested.

15 From stability to compatibility

A recent direction of the project concerns what allows different local stabilizations to coexist. Suppose two observers reconstruct a partial trace in different ways:

$$I_A(T) \neq I_B(T).$$

The difference need not immediately mean that one reconstruction is wrong. The question may instead be whether the two reconstructions can support a common continuation:

$$I_A(T) \;\mathsf{R}\; I_B(T).$$

Here R is not yet a fixed mathematical relation. It marks a research question: what should count as sufficient compatibility for two local constructions to continue interacting?

This suggests a possible direction:

$$\boxed{\text{trace} \to \text{local interpretation} \to \text{comparison} \to \text{compatibility} \to \text{further construction.}}$$

The project does not yet claim a definitive metric, probability or algebra for such compatibility.

16 Projections and equation-theoretic lianas

The latest pages make this compatibility question more concrete through the image of lianas: paths of construction that can be followed, interrupted, projected and narrated. A liana is not itself a finished theory. It can instead provide a way to locate a sequence of operations or distinctions from which a formal theory might be obtained.

The emerging question is whether several distinguishable equation-based theories can be treated as projections of related constructions. A projection need not preserve every distinction. It may stabilize some relations, identify some cases that were previously distinguishable, or make available operations that another projection does not support. The point is not to declare the resulting theories equivalent, but to investigate what remains compatible between them and what their differences reveal.

This gives a possible mathematical entry point:

$$\text{construction} \longrightarrow \text{projection} \longrightarrow \text{equational theory} \longrightarrow \text{comparison of projections.}$$

The arrows are a schematic question, not a claimed universal procedure. The projection, the relevant equations and the notion of compatibility still need to be specified in examples. One useful test would be to exhibit two small projections, identify a distinction preserved by both and another that one of them contracts, and then determine whether a meaningful relation between the two theories remains.

The four roles q, d, p, b and their narrated orientations may help keep track of which local distinction or direction is active during such a construction. They are not yet fixed types or axioms. The lianas are intended to preserve several possible paths of formalization, not to force them into a single global language.

17 Matrix insertions, open completions, and the default ring

A more concrete bridge is emerging between the four-role diagrams and operations on matrices. The point is not merely to decorate a matrix with the labels q, d, p, b, but to let the position and orientation of an insertion contribute to the interpretation of a trace.

One natural reading associates a q-move with inserting an identity-like structure at the bottom of a matrix (or on the left when the construction runs beyond its current extent). This is close to the vertical trace $|$. A complementary insertion, made at the bottom or on the right when the construction extends beyond its current extent, is close to the horizontal trace $-$. The exact conventions still depend on the current orientation and on what counts as overflow. The intended point is that these readings arise from the operations themselves rather than being assigned only after the fact.

The matrices I and J serve as recognizable anchors for two different shapes: I is used for the main diagonal, while J is used here for the antidiagonal. This is a project-specific naming convention. Their role in the construction is not to establish a quantum purification by themselves, but to help describe how a partial matrix representation can be extended or reoriented while retaining a trace of how the extension occurred. The analogy with purification remains an exploratory one: it concerns the gesture of completing a local description in a larger structure, not an asserted identity with the standard quantum-information definition.

The open and completed versions marked $\sharp$ and $\flat$ suggest an alternation of shapes: a rectangle between two rhombi, and a rhombus between two rectangles. The shapes are not simply competing candidates for one canonical geometry. Opening, pointing and completing them can change which shape acts as the local reference. This is a

possible geometric language for what the project calls biextensions of cords: a local shape is read through two neighbouring continuations, and the completion can be approached from either side.

The default ring is the spatial arrangement that keeps these differently oriented extensions available together. It is not intended as an ornamental picture imposed on the matrix operations. It is a way of laying out local insertions, openings and completions so that their transmission paths can be visited and compared. The ring may therefore be a representation of the operations' relations, rather than a geometry assumed in advance.

An earlier geometric intuition sought to synchronize two hyperbolas, one between a horizontal direction and the other between diagonal directions, using an approximation to $\pi/8$. The simpler construction now being explored starts instead from adjacent increasing and decreasing mechanisms. The angle need not be assumed at the outset; it could later be investigated as a feature of a geometric realization, if the dynamics actually produce it.

18 Simple extension: adjacent synchronization

The current "Simple extension" starts from a recurring interruption in a line of thought: a response to "who do you think you are?" may become plausible only after time has passed, after moving, and before death. The roles q, d, p, b are used to narrate these different positions in an interruption, not as fixed types detached from the process.

The notation $:=^{\cdot}$ is being used to mark an identification in which a left part is recognized just after an interruption and a new term emerges on the right. Depending on whether the current narration is q or d, or instead p or b, the new term is imagined above or to the right. Thus even the placement of an equation is part of the represented act of continuation.

A candidate previous time is written

$$|t =_v \frac{tT}{T+1},$$

with the next time also considered through the ordinary successor $t+1$. The exploratory rules couple a time such as t_p to a parameter such as T_d, then investigate the parameter through both a decreasing and an increasing mechanism. Among the proposed relations are

$$T_d^- = T_d \frac{t_q}{t_q+1}, \qquad T_d| = T_d + 1,$$

with adjacent-axis rules linking the other parameters and times. These formulas are not yet a fully specified recurrence: the symbols $|$, $-$, $._p$, $d_|$, and the interruption marker $:=^{\cdot}$ retain orientation information that must be made explicit before numerical iteration is unambiguous.

The guiding idea is that each parameter has an increasing and a decreasing mechanism, while neighbouring axes constrain one another for synchronization. This replaces a preselected geometric synchronization with a dynamical question: what patterns arise from iterating the local rules, and what happens after one cycle through q, d, p, b? Possible outcomes include exact return, periodicity, controlled drift, or merely local compatibility; these are different properties and should not be conflated.

A first mathematical test would specify the full state of the system, its transition rule for each of the four orientations, and the criterion for closure after a cycle. Only then could one ask whether the resulting orbit supports a stable spatial layout, whether a ring is a useful projection of that layout, or whether an angle such as $\pi/8$ emerges in a particular realization. The present construction is a candidate mechanism for generating geometry, not yet a proof that a particular geometry or angle follows from it. The expected interruption loci can also be placed back into the four-role layout. Although one could vary local behavioural reversals, rotational intentions or the direction of local time, the current preference is to explore three "breaths" per word, or three words per breath, rather than four independent carried bits. The position b then acts as a chosen anchor for considering the responsibility involved in continuing in the order *qdpb* or attempting the reverse order *bpdq*.

A k-step version of the familiar identity

$$\left(e^{i\pi/k}\right)^k = e^{i\pi} = -1$$

offers a simple mathematical image for distributing a global transformation across repeated local steps: multiplication composes the steps, while addition of exponents makes the accumulated phase visible. The identity itself is exact; estimating how often analogous torsions occur in localizable processes would require a separate definition of occurrence and measurement. Likewise, the sixteen protofunctions may offer a way to explore two

linked proto-algebras, with one role indexing another, while keeping the operations open to interruption. These remain directions for construction rather than established algebraic results.

19 An anchor and the waves

There is also a more ordinary question underneath the formal one: how does one begin when one does not yet know where one is going?

An anchor suggests stability, but an anchor is useful only in relation to something that moves.

One can think of the project as learning to remain with partial knowledge without turning uncertainty immediately into a defect:

> Nager, c'est quelque part échouer à marcher sur l'eau.

The image is not a theory. It expresses a methodological preference: one may continue with doubts, alongside those who hold certainties, those who do not yet find them, and those who are still recovering their breath.

The project does not seek to abolish uncertainty before construction begins. It asks whether uncertainty can itself become a condition for discovering distinctions.

20 What remains deliberately open

Several questions remain unresolved:

- Which constructions admit useful formalizations?
- Which apparent symmetries are genuine structures and which are consequences of a chosen reading?
- When can several local reconstructions be shown compatible?
- Can interruption delay a choice without merely replacing it by another hidden convention?
- Which mathematical frameworks best preserve the distinctions present in the original constructions?
- What new mathematical phenomena become visible when formalization is deliberately partial?
- Can relations between different formal anchors be characterized without forcing them into one common formal language?
- What would count as "sufficiently" formalized for a construction to become reproducible without pretending that its other possible readings have disappeared?

These are not gaps to be hidden by premature definitions. They are part of the research program.

21 Conclusion: an evolving family of paths

This document should be read neither as a finished theory nor as a collection of unrelated metaphors. It is an attempt to keep visible how gestures, traces, attributions and representations can become locally acceptable through interruptions that remain interruptible.

There is no required trajectory from trace to formalization. A gesture can prompt an attribution; an attribution can shape a representation; the representation can interrupt what had seemed to be the original gesture. Each of these can become a sub-sequence within another interruption.

The project is increasingly less well described by a single route of derivations. The important question is not which continuation should survive, but what can be learned by moving between paths, and which distinctions or limits emerge when local stabilizations are compared. The four-role garden is one recent attempt to make this movement more explicit, without treating its current diagrams as a closed formalism.

Points, boundaries, names, roles, representations and formal languages are not assumed to be unquestioned primitives or final ontologies. They are among the things that can be made, felt, attributed, used, and interrupted. The aim is not to reject rigor, but to let the situated gesture and the tacit work of interpretation remain visible, including when they take mathematical form. A useful next step is to test a small, inspectable example in which the

insertions and the four adjacent transitions can be followed explicitly. One can then ask both what compatibility survives between different projections and whether a cycle produces return, periodicity, controlled drift or only local synchronization. Such an example would offer a mathematical point of entry without making the garden's current imagery into a premature formal system.

This text is an evolving entry point into a larger research notebook.

Note on the writing process

This document is part of an evolving research notebook. Some formulations have been developed and refined through interactive dialogue tools used as exploratory aids. Such tools are treated as instruments for construction and reformulation, not as sources of mathematical authority. The responsibility for the content and its interpretation remains entirely the author's.